



EXERCISES

1. **Coin toss.** Is a coin flip random? Why or why not?
2. **Casino.** A casino claims that its electronic “video roulette” machine is truly random. What should that claim mean?
3. **The lottery.** Many states run lotteries, giving away millions of dollars if you match a certain set of winning numbers. How are those numbers determined? Do you think this method guarantees randomness? Explain.
4. **Games.** Many kinds of games people play rely on randomness. Cite three different methods commonly used in the attempt to achieve this randomness, and discuss the effectiveness of each.
5. **Birth defects.** The American College of Obstetricians and Gynecologists says that out of every 100 babies born in the United States, 3 have some kind of major birth defect. How would you assign random numbers to conduct a simulation based on this statistic?
6. **Colorblind.** By some estimates, about 10% of all males have some color perception defect, most commonly red-green colorblindness. How would you assign random numbers to conduct a simulation based on this statistic?
7. **Geography.** An elementary school teacher with 25 students plans to have each of them make a poster about two different states. The teacher first numbers the states (in alphabetical order, from 1-Alabama to 50-Wyoming), then uses a random number table to decide which states each kid gets. Here are the random digits:

45921 01710 22892 37076

 - a) Which two state numbers does the first student get?
 - b) Which two state numbers go to the second student?
8. **Get rich.** Your state’s BigBucks Lottery prize has reached \$100,000,000, and you decide to play. You have to pick five numbers between 1 and 60, and you’ll win if your numbers match those drawn by the state. You decide to pick your “lucky” numbers using a random number table. Which numbers do you play, based on these random digits?

43680 98750 13092 76561 58712
9. **Play the lottery.** Some people play state-run lotteries by always playing the same favorite “lucky” number. Assuming that the lottery is truly random, is this strategy better, worse, or the same as choosing different numbers for each play? Explain.
10. **Play it again, Sam.** In Exercise 8 you imagined playing the lottery by using random digits to decide what numbers to play. Is this a particularly good or bad strategy? Explain.
11. **Bad simulations.** Explain why each of the following simulations fails to model the real situation properly:
 - a) Use a random integer from 0 through 9 to represent the number of heads when 9 coins are tossed.
 - b) A basketball player takes a foul shot. Look at a random digit, using an odd digit to represent a good shot and an even digit to represent a miss.
 - c) Use random digits from 1 through 13 to represent the denominations of the cards in a five-card poker hand.
12. **More bad simulations.** Explain why each of the following simulations fails to model the real situation:
 - a) Use random numbers 2 through 12 to represent the sum of the faces when two dice are rolled.
 - b) Use a random integer from 0 through 5 to represent the number of boys in a family of 5 children.
 - c) Simulate a baseball player’s performance at bat by letting 0 = an out, 1 = a single, 2 = a double, 3 = a triple, and 4 = a home run.
13. **Wrong conclusion.** A Statistics student properly simulated the length of checkout lines in a grocery store and then reported, “The average length of the line will be 3.2 people.” What’s wrong with this conclusion?
14. **Another wrong conclusion.** After simulating the spread of a disease, a researcher wrote, “24% of the people contracted the disease.” What should the correct conclusion be?
15. **Election.** You’re pretty sure that your candidate for class president has about 55% of the votes in the entire school. But you’re worried that only 100 students will show up to vote. How often will the underdog (the one with 45% support) win? To find out, you set up a simulation.
 - a) Describe how you will simulate a component.
 - b) Describe how you will simulate a trial.
 - c) Describe the response variable.
16. **Two pair or three of a kind?** When drawing five cards randomly from a deck, which is more likely, two pairs or three of a kind? A pair is exactly two of the same denomination. Three of a kind is exactly 3 of the same denomination. (Don’t count three 8’s as a pair—that’s 3 of a kind. And don’t count 4 of the same kind as two pair—that’s 4 of a kind, a very special hand.) How could you simulate 5-card hands? Be careful; once you’ve picked the 8 of spades, you can’t get it again in that hand.
 - a) Describe how you will simulate a component.
 - b) Describe how you will simulate a trial.
 - c) Describe the response variable.
17. **Cereal.** In the chapter’s example, 20% of the cereal boxes contained a picture of Tiger Woods, 30% David Beckham, and the rest Serena Williams. Suppose you buy five boxes of cereal. Estimate the probability that you end up with a complete set of the pictures. Your simulation should have at least 20 runs.
18. **Cereal, again.** Suppose you really want the Tiger Woods picture. How many boxes of cereal do you need to buy to be pretty sure of getting at least one? Your simulation should use at least 10 trials.

19. **Multiple choice.** You take a quiz with 6 multiple choice questions. After you studied, you estimated that you would have about an 80% chance of getting any individual question right. What are your chances of getting them all right? Use at least 20 trials.
20. **Lucky guessing?** A friend of yours who took the multiple choice quiz in Exercise 19 got all 6 questions right, but now claims to have guessed blindly on every question. If each question offered 4 possible answers, do you believe her? Explain, basing your argument on a simulation involving at least 10 trials.
21. **Beat the lottery.** Many states run lotteries to raise money. A Web site advertises that it knows “how to increase YOUR chances of Winning the Lottery.” They offer several systems and criticize others as foolish. One system is called *Lucky Numbers*. People who play the *Lucky Numbers* system just pick a “lucky” number to play, but maybe some numbers are luckier than others. Let’s use a simulation to see how well this system works.
To make the situation manageable, simulate a simple lottery in which a single digit from 0 to 9 is selected as the winning number. Pick a single value to bet, such as 1, and keep playing it over and over. You’ll want to run at least 100 trials. (If you can program the simulations on a computer, run several hundred. Or generalize the questions to a lottery that chooses two- or three-digit numbers—for which you’ll need thousands of trials.)
a) What proportion of the time do you expect to win?
b) Would you expect better results if you picked a “luckier” number, such as 7? (Try it if you don’t know.) Explain.
22. **Random is as random does.** The “beat the lottery” Web site discussed in Exercise 21 suggests that because lottery numbers are random, it is better to select your bet randomly. For the same simple lottery in Exercise 21 (random values from 0 to 9), generate each bet by choosing a separate random value between 0 and 9. Play many games. What proportion of the time do you win?
23. **It evens out in the end.** The “beat the lottery” Web site of Exercise 21 notes that in the long run we expect each value to turn up about the same number of times. That leads to their recommended strategy. First, watch the lottery for a while, recording the winners. Then bet the value that has turned up the least, because it will need to turn up more often to even things out. If there is more than one “rarest” value, just take the lowest one (since it doesn’t matter). Simulating the simplified lottery described in Exercise 21, play many games with this system. What proportion of the time do you win?
24. **Play the winner?** Another strategy for beating the lottery is the reverse of the system described in Exercise 23. Simulate the simplified lottery described in Exercise 21. Each time, bet the number that just turned up. The Web site suggests that this method should do worse. Does it? Play many games and see.
25. **Driving test.** You are about to take the road test for your driver’s license. You hear that only 34% of candidates pass the test the first time, but the percentage rises to 72% on subsequent retests. Estimate the average number of tests drivers take in order to get a license. Your simulation should use at least 20 runs.
26. **Still learning?** As in Exercise 25, assume that your chance of passing the driver’s test is 34% the first time and 72% for subsequent retests. Estimate the percentage of those tested who still do not have a driver’s license after two attempts.
27. **Basketball strategy.** Late in a basketball game, the team that is behind often fouls someone in an attempt to get the ball back. Usually the opposing player will get to shoot foul shots “one and one,” meaning he gets a shot, and then a second shot only if he makes the first one. Suppose the opposing player has made 72% of his foul shots this season. Estimate the number of points he will score in a one-and-one situation.
28. **Blood donors.** A person with type O-positive blood can receive blood only from other type O donors. About 44% of the U.S. population has type O blood. At a blood drive, how many potential donors do you expect to examine in order to get three units of type O blood?
29. **Free groceries.** To attract shoppers, a supermarket runs a weekly contest that involves “scratch-off” cards. With each purchase, customers get a card with a black spot obscuring a message. When the spot is scratched away, most of the cards simply say, “Sorry—please try again.” But during the week, 100 customers will get cards that make them eligible for a drawing for free groceries. Ten of the cards say they may be worth \$200, 10 others say \$100, 20 may be worth \$50, and the rest could be worth \$20. To register those cards, customers write their names on them and put them in a barrel at the front of the store. At the end of the week the store manager draws cards at random, awarding the lucky customers free groceries in the amount specified on their card. The drawings continue until the store has given away more than \$500 of free groceries. Estimate the average number of winners each week.
30. **Find the ace.** A new electronics store holds a contest to attract shoppers. Once an hour someone in the store is chosen at random to play the Music Game. Here’s how it works: An ace and four other cards are shuffled and placed face down on a table. The customer gets to turn cards over one at a time, looking for the ace. The person wins \$100 worth of free CDs or DVDs if the ace is the first card, \$50 if it is the second card, and \$20, \$10, or \$5 if it is the third, fourth, or fifth card chosen. What is the average dollar amount of music the store will give away?
31. **The family.** Many couples want to have both a boy and a girl. If they decide to continue to have children until they have one child of each sex, what would the average family size be? Assume that boys and girls are equally likely.
32. **A bigger family.** Suppose a couple will continue having children until they have at least two children of each sex (two boys *and* two girls). How many children might they expect to have?

33. **Dice game.** You are playing a children's game in which the number of spaces you get to move is determined by the rolling of a die. You must land exactly on the final space in order to win. If you are 10 spaces away, how many turns might it take you to win?
34. **Parcheesi.** You are three spaces from a win in Parcheesi. On each turn, you will roll two dice. To win, you must roll a total of 3 or roll a 3 on one of the dice. How many turns might you expect this to take?
35. **The hot hand.** A basketball player with a 65% shooting percentage has just made 6 shots in a row. The announcer says this player "is hot tonight! She's in the zone!" Assume the player takes about 20 shots per game. Is it unusual for her to make 6 or more shots in a row during a game?
36. **The World Series.** The World Series ends when a team wins 4 games. Suppose that sports analysts consider one team a bit stronger, with a 55% chance to win any individual game. Estimate the likelihood that the underdog wins the series.
37. **Teammates.** Four couples at a dinner party play a board game after the meal. They decide to play as teams of two and to select the teams randomly. All eight people write their names on slips of paper. The slips are thoroughly mixed, then drawn two at a time. How likely is it that every person will be teamed with someone other than the person he or she came to the party with?
38. **Second team.** Suppose the couples in Exercise 37 choose the teams by having one member of each couple write their names on the cards and the other people each pick a card at random. How likely is it that every person will be teamed with someone other than the person he or she came with?
39. **Job discrimination?** A company with a large sales staff announces openings for three positions as regional managers. Twenty-two of the current salespersons apply, 12 men and 10 women. After the interviews, when the company announces the newly appointed managers, all three positions go to women. The men complain of job discrimination. Do they have a case? Simulate a random selection of three people from the applicant pool, and make a decision about the likelihood that a fair process would result in hiring all women.
40. **Cell phones.** A proud legislator claims that your state's new law against talking on a cell phone while driving has reduced cell phone use to less than 12% of all drivers. While waiting for your bus the next morning, you notice that 4 of the 10 people who drive by are using their cell phones. Does this cast doubt on the legislator's figure of 12%? Use a simulation to estimate the likelihood of seeing at least 4 of 10 randomly selected drivers talking on their cell phones if the actual rate of usage is 12%. Explain your conclusion clearly.



JUST CHECKING Answers

1. The component is one game.
2. I'll generate random numbers and assign numbers from 00 to 54 to the home team's winning and from 55 to 99 to the visitors' winning.
3. I'll generate components until one team wins 4 games. I'll record which team wins the series.
4. The response is who wins the series.
5. I'll calculate the proportion of wins by the team that starts at home.