■ Chapter 1 Review

1. (d)

2. (f)

3. (i)

4. (h)

5. (b)

6. (j)

7. (g)

8. (c)

9. (a)

10. (e)

11. **(a)** All reals **(b)** All reals

12. **(a)** All reals **(b)** All reals

13. **(a)** All reals

(b) $g(x) = x^2 + 2x + 1 = (x + 1)^2$.
At $x = -1$, $g(x) = 0$, the function's minimum.
The range is $[0, \infty)$.

14. **(a)** All reals

(b) $(x - 2)^2 \geq 0$ for all x, so $(x - 2)^2 + 5 \geq 5$ for all x.
The range is $[5, \infty)$.

15. **(a)** All reals

(b) $|x| \geq 0$ for all x, so $3|x| \geq 0$ and $3|x| + 8 \geq 8$ for all x. The range is $[8, \infty)$.

16. **(a)** We need $\sqrt{4 - x^2} \geq 0$ for all x, so $4 - x^2 \geq 0$, $4 \geq x^2$, $-2 \leq x \leq 2$. The domain is $[-2, 2]$.

(b) $0 \leq \sqrt{4 - x^2} \leq 2$ for all x, so
$-2 \leq \sqrt{4 - x^2} - 2 \leq 0$ for all x. The range is $[-2, 0]$.

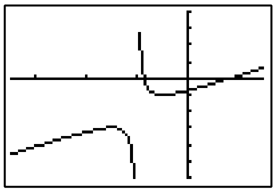
17. **(a)** $f(x) = \dfrac{x}{x^2 - 2x} = \dfrac{x}{x(x - 2)}$. $x \neq 0$ and $x - 2 \neq 0$, $x \neq 2$. The domain is all reals except 0 and 2.

(b) For $x > 2$, $f(x) > 0$ and for $x < 2$, $f(x) < 0$. $f(x)$ does not cross $y = 0$, so the range is all reals except $f(x) = 0$.

18. **(a)** We need
$\sqrt{9-x^2} > 0, 9 - x^2 > 0, 9 > x^2, -3 < x < 3.$
The domain is $(-3, 3)$.

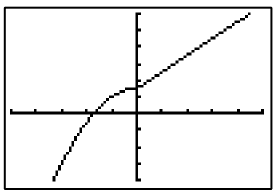
(b) Since $\sqrt{9-x^2} > 0, \dfrac{1}{\sqrt{9-x^2}} > 0$. On the domain $(-3, 3)$, $k(0) = \dfrac{1}{3}$, a minimum, while $k(x)$ approaches ∞ when x approaches both -3 and 3, maximums for $k(x)$. The range is $\left[\dfrac{1}{3}, \infty\right)$.

19. Continuous



$[-7, 3]$ by $[-12, 8]$

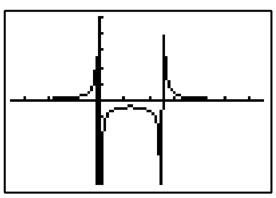
20. Continuous



$[-5, 5]$ by $[-8, 12]$

21. **(a)** $x^2 - 5x \neq 0$, $x(x-5) \neq 0$, so $x \neq 0$ and $x \neq 5$.
We expect vertical asymptotes at $x = 0$ and $x = 5$.

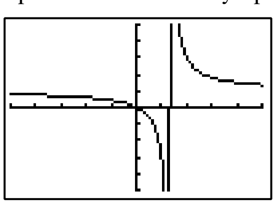
(b) $y = 0$



$[-7, 13]$ by $[-10, 10]$

22. **(a)** $x - 4 \neq 0$, $x \neq 4$, so we expect a vertical asymptote at $x = 4$.

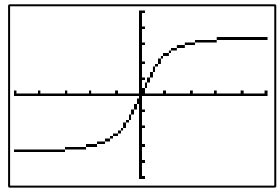
(b) Since $\lim\limits_{x\to\infty} \dfrac{3x}{x-4} = 3$ and $\lim\limits_{x\to-\infty} \dfrac{3x}{x-4} = 3$, we also expect a horizontal asymptote at $y = 3$.



$[-15, 15]$ by $[-15, 15]$

23. **(a)** None

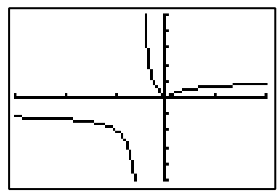
(b) Since $\lim\limits_{x\to\infty} \dfrac{7x}{\sqrt{x^2+10}} = 7$ and $\lim\limits_{x\to-\infty} \dfrac{7x}{\sqrt{x^2+10}} = -7$, we expect horizontal asymptotes at $y = 7$ and $y = -7$.



$[-15, 15]$ by $[-10, 10]$

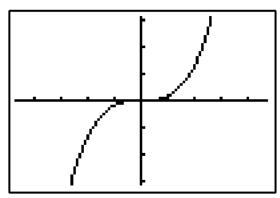
24. **(a)** $x + 1 \neq 0$, $x \neq -1$, so we expect a vertical asymptote at $x = -1$.

(b) $\lim\limits_{x\to\infty} \dfrac{|x|}{x+1} = 1$ and $\lim\limits_{x\to-\infty} \dfrac{|x|}{x+1} = -1$, so we can expect horizontal asymptotes at $y = 1$ and $y = -1$.



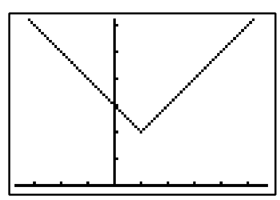
$[-6, 4]$ by $[-5, 5]$

25. $(-\infty, \infty)$



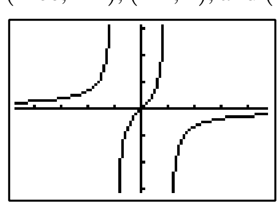
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

26. $|x - 1| = 0$ when $x = 1$, which is where the function's minimum occurs. y increases over the interval $[1, \infty)$. (Over the interval $(-\infty, 1]$, it is decreasing.)



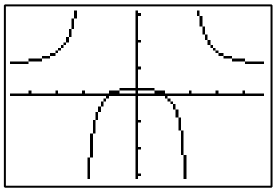
$[-3.7, 5.7]$ by $[0, 6.2]$

27. As the graph illustrates, y is increasing over the intervals $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$.



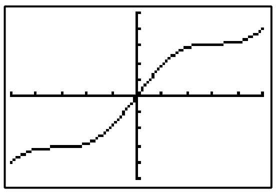
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

28. As the graph illustrates, y is increasing over the intervals $(-\infty, -2)$ and $(-2, 0]$.



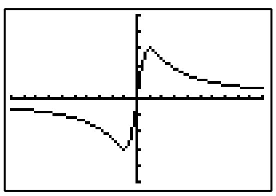
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

29. $-1 \le \sin x \le 1$, but $-\infty < x < \infty$, so $f(x)$ is not bounded.



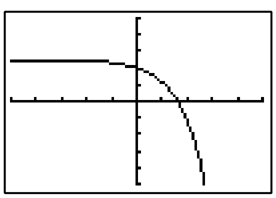
$[-5, 5]$ by $[-5, 5]$

30. $g(x) = 3$ at $x = 1$, a maximum and $g(x) = -3$, a minimum, at $x = -1$. It is bounded.



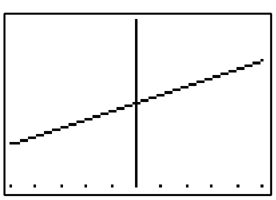
$[-10, 10]$ by $[-5, 5]$

31. $e^x > 0$ for all x, so $-e^x < 0$ and $5 - e^x < 5$ for all x. $h(x)$ is bounded above.



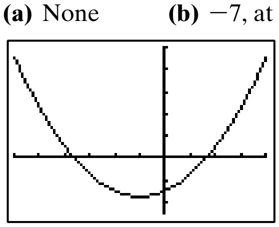
$[-5, 5]$ by $[-10, 10]$

32. The function is linear with slope $\frac{1}{1000}$ and y-intercept 1000. Thus $k(x)$ is not bounded.



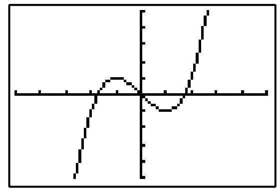
$[-5, 5]$ by $[-999.99, 1000.01]$

33. **(a)** None **(b)** -7, at $x = -1$



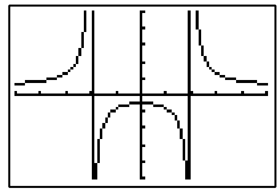
$[-6, 4]$ by $[-10, 20]$

34. **(a)** 2, at $x = -1$ **(b)** -2, at $x = 1$



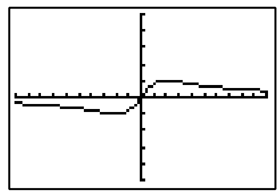
$[-5, 5]$ by $[-10, 10]$

35. **(a)** -1, at $x = 0$ **(b)** None



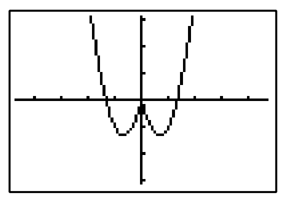
$[-5, 5]$ by $[-10, 10]$

36. **(a)** 1, at $x = 2$ **(b)** -1, at $x = -2$



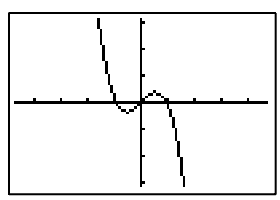
$[-10, 10]$ by $[-5, 5]$

37. The function is even since it is symmetrical about the y-axis.



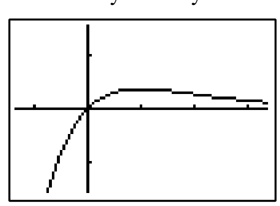
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

38. Since the function is symmetrical about the origin, it is odd.



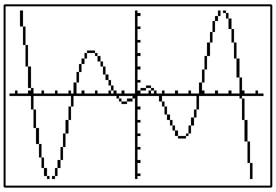
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

39. Since no symmetry is exhibited, the function is neither.



$[-1.35, 3.35]$ by $[-1.55, 1.55]$

40. Since the function is symmetrical about the origin, it is odd.



$[-9.4, 9.4]$ by $[-6.2, 6.2]$

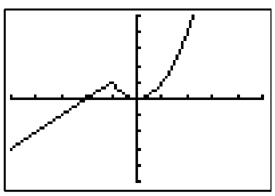
41. $x = 2y + 3, 2y = x - 3, y = \dfrac{x-3}{2}$, so $f^{-1}(x) = \dfrac{x-3}{2}$.

42. $x = \sqrt[3]{y-8}, x^3 = y - 8, y = x^3 + 8$, so $f^{-1}(x) = x^3 + 8$.

43. $x = \dfrac{2}{y}, xy = 2, y = \dfrac{2}{x}$, so $f^{-1}(x) = \dfrac{2}{x}$.

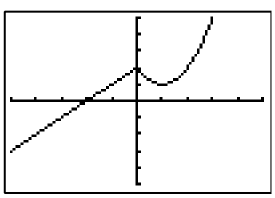
44. $x = \dfrac{6}{y+4}, (y+4)x = 6, xy + 4x = 6, xy = 6 - 4x,$ $y = \dfrac{6-4x}{x}$, so $f^{-1}(x) = \dfrac{6}{x} - 4$.

45.



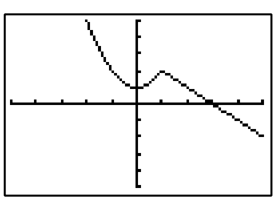
$[-5, 5]$ by $[-5, 5]$

46.



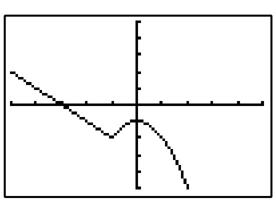
$[-5, 5]$ by $[-5, 5]$

47.



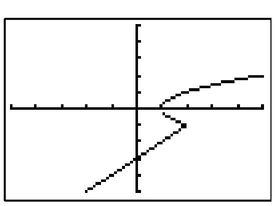
$[-5, 5]$ by $[-5, 5]$

48.



$[-5, 5]$ by $[-5, 5]$

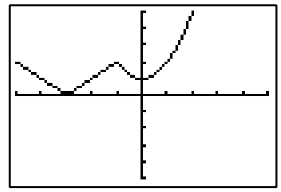
49.



$[-5, 5]$ by $[-5, 5]$

50. No

51.



$[-5, 5]$ by $[-5, 5]$

52. $f(x) = \begin{cases} x + 3 \text{ if } x \le -1 \\ x^2 + 1 \text{ if } x > -1 \end{cases}$

53. $(f \circ g)(x) = f(g(x)) = f(x^2 - 4) = \sqrt{x^2 - 4}$. Since $x^2 - 4 \ge 0, x^2 \ge 4, x \le -2$ or $x \ge 2$. The domain is $(-\infty, -2] \cup [2, \infty)$.

54. $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 4 = x - 4$. Since $\sqrt{x} \ge 0, x \ge 0$. The domain is $[0, \infty)$.

55. $(f \cdot g)(x) = f(x) \cdot g(x) = \sqrt{x} \cdot (x^2 - 4)$. Since $\sqrt{x} \ge 0$, the domain is $[0, \infty)$.

56. $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x}}{x^2 - 4}$ Since $x^2 - 4 \ne 0$, $(x+2)(x-2) \ne 0, x \ne -2, x \ne 2$. Also since $\sqrt{x} \ge 0, x \ge 0$. The domain is $[0, 2) \cup (2, \infty)$.

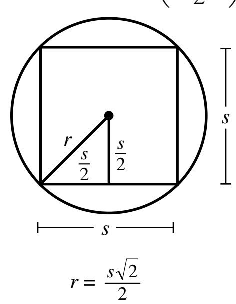
57. $\lim_{x\to\infty} \sqrt{x} = \infty$. (Large negative values are not in the domain.)

58. $\lim_{x\to\pm\infty} \sqrt{x^2 - 4} = \infty$. (The graph resembles the line $y = x$.)

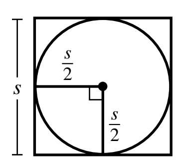
59. $r^2 = \left(\dfrac{s}{2}\right)^2 + \left(\dfrac{s}{2}\right)^2 = \dfrac{2s^2}{4}, \ r = \sqrt{\dfrac{2s^2}{4}} = \dfrac{s\sqrt{2}}{2}$.

The area of the circle is

$A = \pi r^2 = \pi\left(\dfrac{s\sqrt{2}}{2}\right)^2 = \dfrac{2\pi s^2}{4} = \dfrac{\pi s^2}{2}$

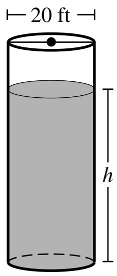


60. $A = \pi r^2 = \pi\left(\dfrac{s}{2}\right)^2 = \dfrac{\pi s^2}{4}$

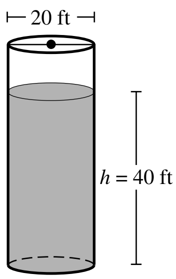


61. $d = 2r$, $r = \dfrac{d}{2}$, so the radius of the tank is 10 feet.

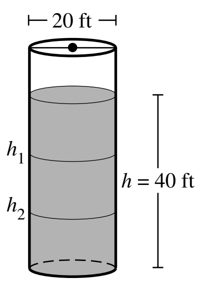
Volume is $V = \pi r^2 \cdot h = \pi(10)^2 \cdot h = 100\pi h$



62. The volume of oil in the tank is the amount of original oil $(\pi r^2 h)$ minus the amount of oil drained.
$V = \pi r^2 h - 2t = \pi(10)^2(40) - 2t = 4000\pi - 2t$

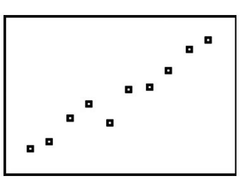


63. Since $V = 4000\pi - 2t$, we know that $\pi r^2 h = 4000\pi - 2t$. In this case, $r = 10'$, so
$100\pi h = 4000\pi - 2t$, $h = \dfrac{4000\pi - 2t}{100\pi} = 40 - \dfrac{t}{50\pi}$



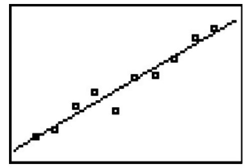
64. Since the depth of the tank is decreasing by 2 feet per hour, we know that the tank is losing a total volume of $V = \pi r^2 h = \pi(10)^2(2) = 200\pi$ cubic feet per hour. The volume of remaining oil in the tank is the amount of original oil subtracting the amount which has been drained, or $V = 4000\pi - 200\pi t$. This is a significantly higher loss than our solution in #68!

65. (a)



[4, 15] by [940, 1700]

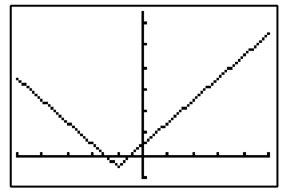
(b) The regression line is $y = 61.133x + 725.333$.



[4, 15] by [940, 1700]

(c) $61.133(20) + 725.333 \approx 1948$ (thousands of barrels)

66. (a)

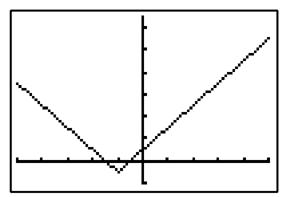


$[-10, 10]$ by $[-2, 18]$

(b) The linear model would eventually intersect the x-axis, which would represent a swimmer covering 100 meters in a time of 0.00. This is clearly impossible.

(c) Based on the data, 52 seconds represents the limit of women's capability in this race. The addition of future data could determine a different model.

(d) The regression curve is $y = (97.100)(0.9614^x)$.



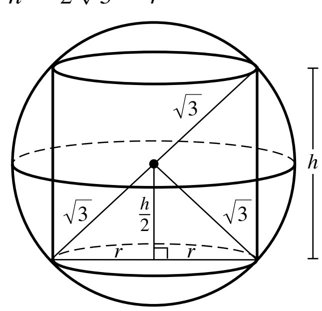
$[-10, 10]$ by $[-2, 18]$

(e) $(97.100)(0.9614^{108}) \approx 1.38$. Add 52 to find the projected winning time in 2008: $1.38 + 52 = 53.38$ seconds.

67. (a) $r^2 + \left(\dfrac{h}{2}\right)^2 = (\sqrt{3})^2$,

$\dfrac{h^2}{4} = 3 - r^2$, $h^2 = 12 - 4r^2$, $h = \sqrt{12 - 4r^2}$,

$h = 2\sqrt{3 - r^2}$

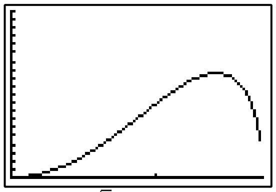


$h = 2\sqrt{3 - r^2}$

(b) $V = \pi r^2 h = (\pi r^2)(2\sqrt{3 - r^2}) = 2\pi r^2\sqrt{3 - r^2}$

(c) Since $\sqrt{3 - r^2} \geq 0$, $3 - r^2 \geq 0$
$3 \geq r^2$, $-\sqrt{3} \leq r \leq \sqrt{3}$.
However, $r < 0$ are invalid values, so the domain is $[0, \sqrt{3}]$.

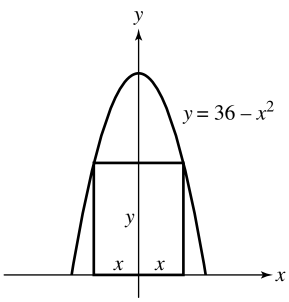
(d)



$[0, \sqrt{3}]$ by $[0, 20]$

(e) 12.57 in^3

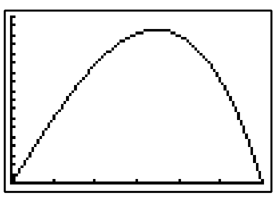
68.



(a) $A = 2xy = 2x(36 - x^2) = 72x - 2x^3$

(b) $36 - x^2 \geq 0, (6 - x)(6 + x) \geq 0, -6 \leq x \leq 6.$ However, $x < 0$ are invalid values, so the domain is $[0, 6]$.

(c)



$[0, 6]$ by $[0, 180]$

(d) The maximum area occurs when $x \approx 3.46$, or an area of approximately 166.28 square units.