CHAPTER 1 Key Ideas

Properties, Theorems, and Formulas

Procedures

CHAPTER 1 Review Exercises

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–10, match the graph with the corresponding function (a)–(j) from the list below. Use your knowledge of function behavior, *not* your grapher.

(a) $f(x) = x^2 - 1$ **(b)** $f(x) = x^2 + 1$

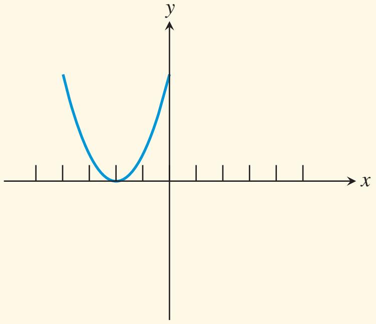
(c) $f(x) = (x - 2)^2$ **(d)** $f(x) = (x + 2)^2$

(e) $f(x) = \dfrac{x - 1}{2}$ **(f)** $f(x) = |x - 2|$

(g) $f(x) = |x + 2|$ **(h)** $f(x) = -\sin x$

(i) $f(x) = e^x - 1$ **(j)** $f(x) = 1 + \cos x$

1.



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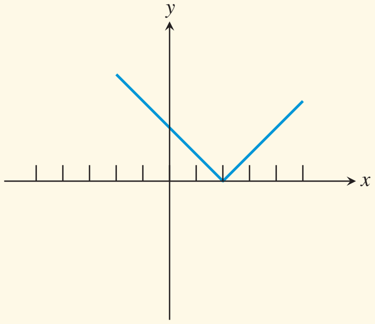

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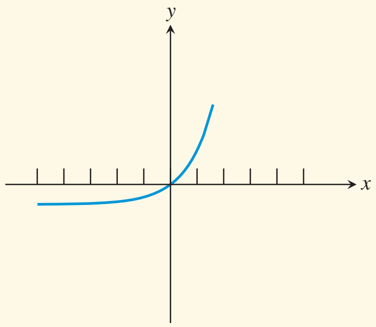

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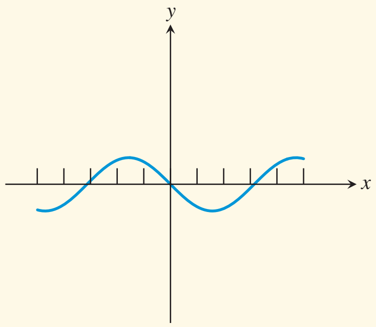

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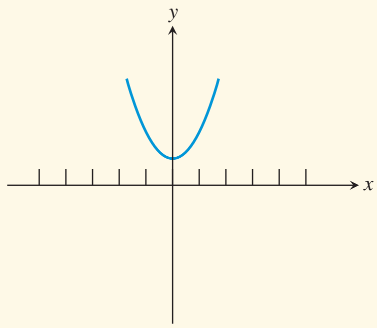

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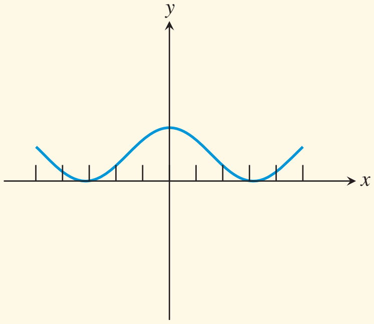

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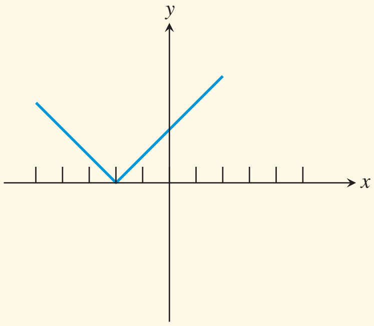

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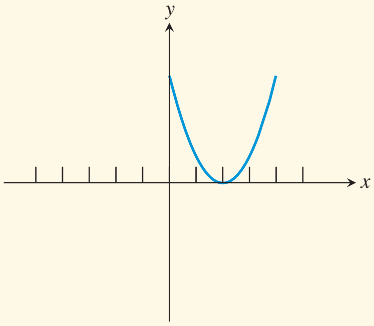

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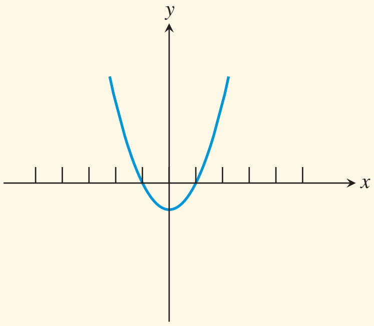

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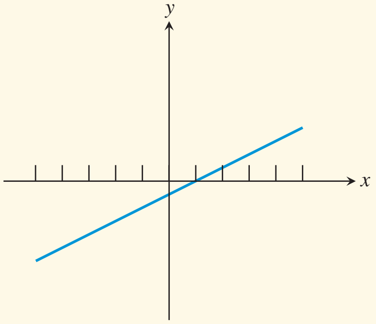


In Exercises 11–18, find (a) the domain and (b) the range of the function.

11. $g(x) = x^3$

12. $f(x) = 35x - 602$

13. $g(x) = x^2 + 2x + 1$

14. $h(x) = (x - 2)^2 + 5$

15. $g(x) = 3|x| + 8$

16. $k(x) = -\sqrt{4 - x^2}$

17. $f(x) = \dfrac{x}{x^2 - 2x}$

18. $k(x) = \dfrac{1}{\sqrt{9 - x^2}}$

In Exercises 19 and 20, graph the function, and state whether the function is continuous at $x = 0$. If it is discontinuous, state whether the discontinuity is removable or nonremovable.

19. $f(x) = \dfrac{x^2 - 3}{x + 2}$

20. $k(x) = \begin{cases} 2x + 3 \text{ if } x > 0 \\ 3 - x^2 \text{ if } x \le 0 \end{cases}$

In Exercises 21–24, find all (a) vertical asymptotes and (b) horizontal asymptotes of the graph of the function. Be sure to state your answers as equations of lines.

21. $y = \dfrac{5}{x^2 - 5x}$

22. $y = \dfrac{3x}{x - 4}$

23. $y = \dfrac{7x}{\sqrt{x^2 + 10}}$

24. $y = \dfrac{|x|}{x + 1}$

In Exercises 25–28, graph the function and state the intervals on which the function is *increasing*.

25. $y = \dfrac{x^3}{6}$

26. $y = 2 + |x - 1|$

27. $y = \dfrac{x}{1 - x^2}$

28. $y = \dfrac{x^2 - 1}{x^2 - 4}$

In Exercises 29–32, graph the function and tell whether the function is bounded above, bounded below, or bounded.

29. $f(x) = x + \sin x$

30. $g(x) = \dfrac{6x}{x^2 + 1}$

31. $h(x) = 5 - e^x$

32. $k(x) = 1000 + \dfrac{x}{1000}$

In Exercises 33–36, use a grapher to find all (a) relative maximum values and (b) relative minimum values of the function. Also state the value of x at which each relative extremum occurs.

33. $y = (x + 1)^2 - 7$

34. $y = x^3 - 3x$

35. $y = \dfrac{x^2 + 4}{x^2 - 4}$

36. $y = \dfrac{4x}{x^2 + 4}$

In Exercises 37–40, graph the function and state whether the function is odd, even, or neither.

37. $y = 3x^2 - 4|x|$

38. $y = \sin x - x^3$

39. $y = \dfrac{x}{e^x}$

40. $y = x\cos(x)$

In Exercises 41–44, find a formula for $f^{-1}(x)$.

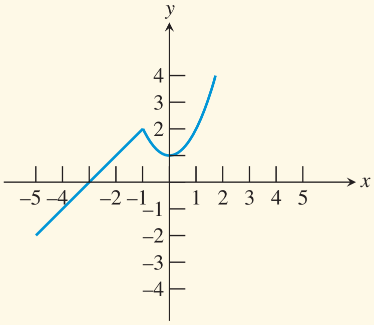
41. $f(x) = 2x + 3$

42. $f(x) = \sqrt[3]{x - 8}$

43. $f(x) = \dfrac{2}{x}$

44. $f(x) = \dfrac{6}{x + 4}$

Exercises 45–52 refer to the function $y = f(x)$ whose graph is given below.



45. Sketch the graph of $y = f(x) - 1$.

46. Sketch the graph of $y = f(x - 1)$.

47. Sketch the graph of $y = f(-x)$.

48. Sketch the graph of $y = -f(x)$.

49. Sketch a graph of the inverse relation.

50. Does the inverse relation define y as a function of x?

51. Sketch a graph of $y = |f(x)|$.

52. Define f algebraically as a piecewise function. [*Hint*: the pieces are translations of two of our "basic" functions.]

In Exercises 53–58, let $f(x) = \sqrt{x}$ and let $g(x) = x^2 - 4$.

53. Find an expression for $(f \circ g)(x)$ and give its domain.

54. Find an expression for $(g \circ f)(x)$ and give its domain.

55. Find an expression for $(fg)(x)$ and give its domain.

56. Find an expression for $\left(\frac{f}{g}\right)(x)$ and give its domain.

57. Describe the end behavior of the graph of $y = f(x)$.

58. Describe the end behavior of the graph of $y = f(g(x))$.

In Exercises 59–64, write the specified quantity as a function of the specified variable. Remember that drawing a picture will help.

59. Square Inscribed in a Circle A square of side s is inscribed in a circle. Write the area of the circle as a function of s.

60. Circle Inscribed in a Square A circle is inscribed in a square of side s. Write the area of the circle as a function of s.

61. Volume of a Cylindrical Tank A cylindrical tank with diameter 20 feet is partially filled with oil to a depth of h feet. Write the volume of oil in the tank as a function of h.

62. Draining a Cylindrical Tank A cylindrical tank with diameter 20 feet is filled with oil to a depth of 40 feet. The oil begins draining at a constant rate of 2 cubic feet per second. Write the volume of the oil remaining in the tank t seconds later as a function of t.

63. Draining a Cylindrical Tank A cylindrical tank with diameter 20 feet is filled with oil to a depth of 40 feet. The oil begins draining at a constant rate of 2 cubic feet per second. Write the depth of the oil remaining in the tank t seconds later as a function of t.

64. Draining a Cylindrical Tank A cylindrical tank with diameter 20 feet is filled with oil to a depth of 40 feet. The oil begins draining so that the depth of oil in the tank decreases at a constant rate of 2 feet per hour. Write the volume of oil remaining in the tank t hours later as a function of t.

65. U.S. Crude Oil Imports The imports of crude oil to the United States from Canada in the years 2000–2008 (in thousands of barrels per day) are given in Table 1.15.

Table 1.15 Crude Oil Imports from Canada

Year	Barrels/day × 1000
2000	1267
2001	1297
2002	1418
2003	1535
2004	1587
2005	1602
2006	1758
2007	1837
2008	1869

Source: Energy Information Administration, Petroleum Supply Monthly, as reported in The World Almanac and Book of Facts 2009.

(a) Sketch a scatter plot of import numbers in the right-hand column (y) as a function of years since 2000 (x).

(b) Find the equation of the regression line and superimpose it on the scatter plot.

(c) Based on the regression line, approximately how many thousands of barrels of oil would the United States import from Canada in 2015?

66. The winning times in the women's 100-meter freestyle event at the Summer Olympic Games since 1956 are shown in Table 1.16.

Table 1.16 Women's 100-Meter Freestyle

Year	Time	Year	Time
1956	62.0	1984	55.92
1960	61.2	1988	54.93
1964	59.5	1992	54.64
1968	60.0	1996	54.50
1972	58.59	2000	53.83
1976	55.65	2004	53.84
1980	54.79	2008	53.12

Source: The World Almanac and Book of Facts 2009.

(a) Sketch a scatter plot of the times (y) as a function of the years (x) beyond 1900. (The values of x will run from 56 to 108.)

(b) Explain why a linear model cannot be appropriate for these times over the long term.

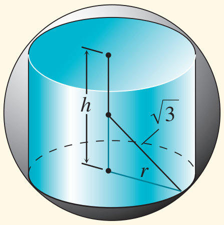
(c) The points appear to be approaching a horizontal asymptote of $y = 52$. What would this mean about the times in this Olympic event?

(d) Subtract 52 from all the times so that they will approach an asymptote of $y = 0$. Redo the scatter plot with the new y-values. Now find the *exponential* regression curve and superimpose its graph on the vertically shifted scatter plot.

(e) According to the regression curve, what will be the winning time in the women's 100-meter freestyle event at the 2016 Olympics?

67. Inscribing a Cylinder Inside a Sphere A right circular cylinder of radius r and height h is inscribed inside a sphere of radius $\sqrt{3}$ inches.

(a) Use the Pythagorean Theorem to write h as a function of r.



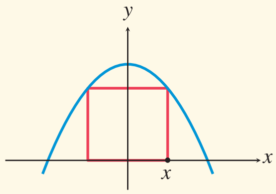
(b) Write the volume V of the cylinder as a function of r.

(c) What values of r are in the domain of V?

(d) Sketch a graph of $V(r)$ over the domain $[0, \sqrt{3}]$.

(e) Use your grapher to find the maximum volume that such a cylinder can have.

68. Inscribing a Rectangle Under a Parabola A rectangle is inscribed between the x-axis and the parabola $y = 36 - x^2$ with one side along the x-axis, as shown in the figure below.



(a) Let x denote the x-coordinate of the point highlighted in the figure. Write the area A of the rectangle as a function of x.

(b) What values of x are in the domain of A?

(c) Sketch a graph of $A(x)$ over the domain.

(d) Use your grapher to find the maximum area that such a rectangle can have.