Chapter 1
Functions and Graphs

■ Section 1.1 Modeling and Equation Solving

Exploration 1

1. $k = \frac{d}{m} = \frac{100 - 25}{100} = \frac{75}{100} = 0.75$

2. $t = 6.5\% + 0.5\% = 7\%$ or 0.07

3. $m = \frac{d}{k}, s = d + td$

$s = pm$

$p = \frac{s}{m} = \frac{d + td}{\frac{d}{k}} = \frac{d + td}{1} \cdot \frac{k}{d} = \frac{d(1 + t)}{1} \cdot \frac{k}{d}$

$= \frac{k(1 + t)}{1} = (0.75)(1.07) = 0.8025$

4. Yes, because $\$36.99 \times 0.8025 = \$29.68$.

5. $\$100 \div 0.8025 = \$124.61$

Exploration 2

1. Because the linear model maintains a constant positive slope, it will eventually reach the point where 100% of the prisoners are female. It will then continue to rise, giving percentages above 100%, which are impossible.

2. Yes, because 2009 is still close to the data we are modeling. We would have much less confidence in the linear model for predicting the percentage 25 years from 2000.

3. One possible answer: Males are heavily dominant in violent crime statistics, while female crimes tend to be property crimes like burglary or shoplifting. Since property crimes rates are senstive to economic conditions, a statistician might look for adverse economic factors in 1990, especially those that would affect people near or below the poverty level.

4. Yes. Table 1.1 shows that the minimum wage worker had less purchasing power in 1990 than in any other year since 1955, which gives some evidence of adverse economic conditions among lower-income Americans that year. Nonetheless, a careful sociologist would certainly want to look at other data before claiming a connection between this statistic and the female crime rate.

Quick Review 1.1

1. $(x + 4)(x - 4)$

2. $(x + 5)(x + 5)$

3. $(9y + 2)(9y - 2)$

4. $3x(x^2 - 5x + 6) = 3x(x - 2)(x - 3)$

5. $(4h^2 + 9)(4h^2 - 9) = (4h^2 + 9)(2h + 3)(2h - 3)$

6. $(x + h)(x + h)$

7. $(x + 4)(x - 1)$

8. $x^2 - 3x + 4$

9. $(2x - 1)(x - 5)$

10. $(x^2 + 5)(x^2 - 4) = (x^2 + 5)(x + 2)(x - 2)$

Section 1.1 Exercises

1. (d) (q)

2. (f) (r)

3. (a) (p)

4. (h) (o)

5. (e) (l)

6. (b) (s)

7. (g) (t)

8. (j) (k)

9. (i) (m)

10. (c) (n)

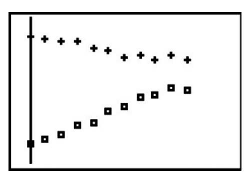
11. **(a)** The percentage increased from 1954 to 1999 and then decreased slightly from 1999 to 2004.

(b) The greatest increase occurred between 1974 and 1979.

12. **(a)** Except for some minor fluctuations, the percentage has been decreasing overall.

(b) The greatest decrease occurred between 1979 and 1984.

13. Women (□), Men (+)



[–5, 55] by [23, 92]

14. Vice versa: The female percentages are increasing faster than the male percentages are decreasing.

15. To find the equation, first find the slope.

Women: Slope $= \frac{\text{change in } y}{\text{change in } x} = \frac{58.5 - 32.3}{1999 - 1954} = \frac{26.2}{45}$ $= 0.582$. The y-intercept is 32.3, so the equation of the line is $y = 0.582x + 32.3$.

Men: Slope $= \frac{74.0 - 83.5}{1999 - 1954} = \frac{-9.5}{45} = -0.211$. The y-intercept is 83.5, so the equation of the line is $y = -0.211x + 83.5$.

In both cases, x represents the number of years after 1954.

16. 2009 is 55 years since 1954, so $x = 55$.
Women: $y = (0.582)(55) + 32.3 \approx 64.3\%$
Men: $y = (-0.211)(55) + 83.5 \approx 71.9\%$

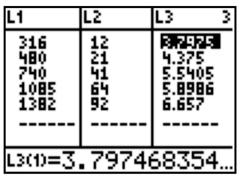
17. For the percentages to be the same, we need to set the two equations equal to each other.

$$\begin{aligned} 0.582x + 32.3 &= -0.211x + 83.5 \\ 0.793x &= 51.2 \\ x &\approx 64.6 \end{aligned}$$

So, approximately 65 years after 1954 (2018), the models predict that the percentages will be about the same. To check:

Males: $y = (-0.211)(65) + 83.5 \approx 69.9\%$
Females: $y = (0.582)(65) + 32.3 \approx 69.9\%$

18. The linear equations will eventually give percentages above 100% for women and below 0% for men, neither of which is possible.

19. 

20. Let h be the height of the rectangular cake in inches.
The volume of the rectangular cake is
$V_1 = 9 \cdot 13 \cdot h = 117h \text{ in.}^3$
The volume of the round cake is
$V_2 = \pi(4)^2(2h) \approx 3.14 \cdot 16 \cdot 2h = 100.48h \text{ in.}^3$
The rectangular cake gives a greater amount of cake for the same price.

21. Because all stepping stones have the same thickness, what matters is area.
The area of a square stepping stone is
$A_1 = 12 \cdot 12 = 144 \text{ in.}^2$
The area of a round stepping stone is

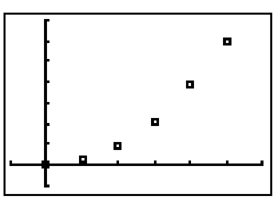
$$A_2 = \pi\left(\frac{13}{2}\right)^2 \approx 3.14(6.5)^2 = 132.665 \text{ in.}^2$$

The square stones give a greater amount of rock for the same price.

22. **(a)** $t = \frac{1}{4}\sqrt{180} \approx 3.35$ sec

(b) $d = 16(12.5)^2 = 2500$ ft

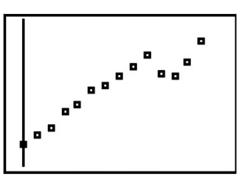
23. A scatter plot of the data suggests a parabola with its vertex at the origin.



[–1, 6] by [–5, 35]

The model $y = 1.2t^2$ fits the data.

24. **(a)**

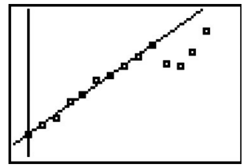


[–1, 15] by [400, 750]

(b) To find the equation, first find the slope:

$$\text{Slope} = \frac{666.1 - 452.2}{2000 - 1991} = \frac{213.9}{9} \approx 23.77.$$

The y-intercept is 452.3, so the equation of the line is $y = 23.77x + 452.3$.



[–1, 15] by [400, 750]

(c) To find the year the number of passengers should reach 900, let $y = 900$, and solve the equation for x. $900 = 23.77x + 452.3$; $x \approx 19$, so by the model, the number of passengers should reach 900 million by 2010 (1991 + 19).

(d) The terrorist attacks on September 11, 2001, caused a major disruption in American air traffic from which the airline industry was slow to recover.

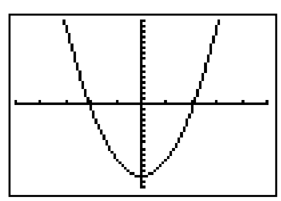
25. The lower line shows the minimum salaries, since they are lower than the average salaries.

26. The points that show the 1990 salaries are the Year 10 points. Both graphs show unprecedented increases in that year. Note: At year 10 the minimum salary jumps, but at year 11 the average salary jumps.

27. The 1995 points are third from the right, Year 15, on both graphs. There is a clear drop in the average salary right after the 1994 strike.

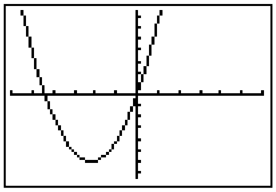
28. One possible answer: **(a)** The players will be happy to see the average salary continue to rise at this rate. The discrepancy between the minimum salary and the average salary will not bother baseball players like it would factory workers, because they are happy to be in the major leagues with the chance to become a star. **(b)** The team owners are not happy with this graph because it shows that their top players are being paid more and more money, forcing them to pay higher salaries to be competitive. This benefits the wealthiest owners. **(c)** Fans are unhappy with the higher ticket prices and with the emphasis on money in baseball rather than team loyalty. Fans of less wealthy teams are unhappy that rich owners are able to pay high salaries to build super-teams filled with talented free agents.

29. Adding $2v^2 + 5$ to both sides gives $3v^2 = 13$. Divide both sides by 3 to get $v^2 = \frac{13}{3}$, so $v = \pm\sqrt{\frac{13}{3}}$.
$3v^2 = 13$ is equivalent to $3v^2 - 13 = 0$. The graph of $y = 3v^2 - 13$ is zero for $v \approx -2.08$ and for $v \approx 2.08$.



[–5, 5] by [–15, 15]

30. $x + 11 = \pm 11$ so $x = -11 \pm 11$, which gives $x = -22$ or $x = 0$.
$(x + 11)^2 = 121$ is equivalent to $(x + 11)^2 - 121 = 0$. The graph of $y = (x + 11)^2 - 121$ is zero for $x = -22$ and for $x = 0$.

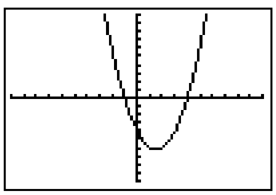


[–30, 30] by [–150, 150]

31.

$$2x^2 - 5x + 2 = x^2 - 5x + 6 + 3x$$
$$x^2 - 3x - 4 = 0$$
$$(x - 4)(x + 1) = 0$$
$$x - 4 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 4 \quad \text{or} \quad x = -1$$

$2x^2 - 5x + 2 = (x - 3)(x - 2) + 3x$ is equivalent to $2x^2 - 8x + 2 - (x - 3)(x - 2) = 0$. The graph of $y = 2x^2 - 8x + 2 - (x - 3)(x - 2)$ is zero for $x = -1$ and for $x = 4$.

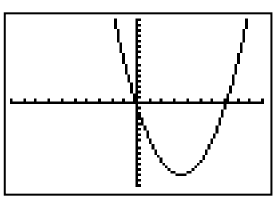


[–10, 10] by [–10, 10]

32.

$$x^2 - 7x = \frac{3}{4}$$
$$x^2 - 7x + \left(-\frac{7}{2}\right)^2 = 0.75 + \left(-\frac{7}{2}\right)^2$$
$$(x - 3.5)^2 = 0.75 + 12.25$$
$$x - 3.5 = \pm\sqrt{13}$$
$$x = 3.5 \pm \sqrt{13}$$

The graph of $y = x^2 - 7x - \frac{3}{4}$ is zero for $x \approx -0.11$ and for $x \approx 7.11$.

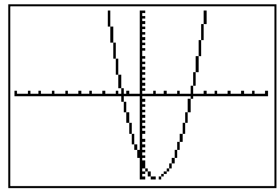


[–10, 10] by [–15, 15]

33. Rewrite as $2x^2 - 5x - 12 = 0$; the left side factors to $(2x + 3)(x - 4) = 0$:

$$2x + 3 = 0 \quad \text{or} \quad x - 4 = 0$$
$$2x = -3 \qquad x = 4$$
$$x = -1.5$$

The graph of $y = 2x^2 - 5x - 12$ is zero for $x = -1.5$ and for $x = 4$.

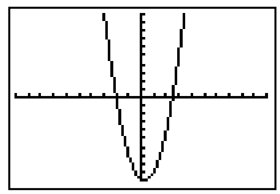


[–10, 10] by [–15, 15]

34. Rewrite as $2x^2 - x - 10 = 0$; the left side factors to $(x + 2)(2x - 5) = 0$:

$$x + 2 = 0 \quad \text{or} \quad 2x - 5 = 0$$
$$x = -2 \qquad 2x = 5$$
$$x = 2.5$$

The graph of $y = 2x^2 - x - 10$ is zero for $x = -2$ and for $x = 2.5$.

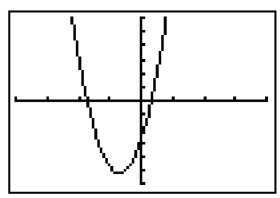


[–10, 10] by [–10, 10]

35. $x^2 + 7x - 14 = 0$, so $a = 1, b = 7$, and $c = -14$:

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-14)}}{2(1)} = \frac{-7 \pm \sqrt{105}}{2}$$
$$= -\frac{7}{2} \pm \frac{1}{2}\sqrt{105}$$

The graph of $y = x^2 + 7x - 14$ is zero for $x \approx -8.62$ and for $x \approx 1.62$.

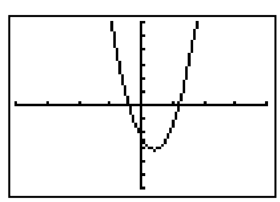


[–20, 20] by [–30, 30]

36. $x^2 - 4x - 12 = 0$, so $a = 1, b = -4$, and $c = -12$:

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)} = \frac{4 \pm \sqrt{64}}{2}$$
$$= 2 \pm \frac{8}{2} = 2 \pm 4$$
$$x = -2 \text{ or } x = 6$$

The graph of $y = x^2 - 4x - 12$ is zero for $x = -2$ and for $x = 6$.

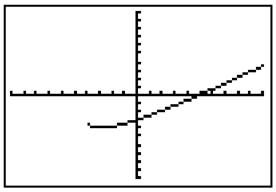


[–20, 20] by [–30, 30]

37. Change to $x^2 - 2x - 15 = 0$ (see below); this factors to $(x + 3)(x - 5) = 0$, so $x = -3$ or $x = 5$. Substituting the first of these shows that it is extraneous.

$$x + 1 = 2\sqrt{x + 4}$$
$$(x + 1)^2 = 2^2(\sqrt{x + 4})^2$$
$$x^2 + 2x + 1 = 4x + 16$$
$$x^2 - 2x - 15 = 0$$

The graph of $y = x + 1 - 2\sqrt{x + 4}$ is zero for $x = 5$.



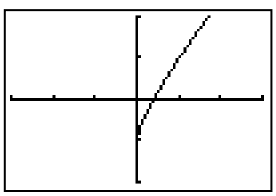
[–10, 10] by [–10, 10]

38. Change to $x^2 - 3x + 1 = 0$ (see below); then $x = \dfrac{3 \pm \sqrt{9 - 4}}{2} = \dfrac{3}{2} \pm \dfrac{1}{2}\sqrt{5}$, so $x = \dfrac{3}{2} - \dfrac{\sqrt{5}}{2}$. Substituting the second of these shows that it is extraneous.

$$\sqrt{x} = 1 - x$$
$$(\sqrt{x})^2 = (1 - x)^2$$
$$x = 1 - 2x + x^2$$
$$0 = x^2 - 3x + 1$$

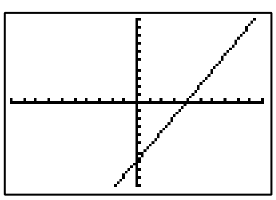
$\sqrt{x} + x = 1$ is equivalent to $x + \sqrt{x} - 1 = 0$.

The graph of $y = x + \sqrt{x} - 1$ is zero for $x \approx 0.38$.



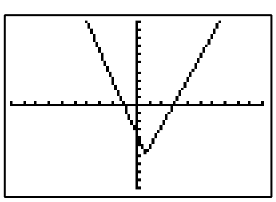
[–3, 3] by [–2, 2]

39. $x \approx 3.91$



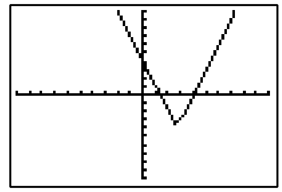
[−10, 10] by [−10, 10]

40. $x \approx -1.09$ or $x \approx 2.86$



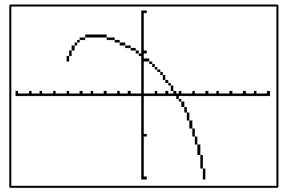
[−10, 10] by [−10, 10]

41. $x \approx 1.33$ or $x = 4$



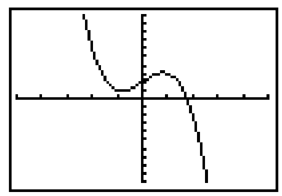
[−10, 10] by [−10, 10]

42. $x \approx 2.66$



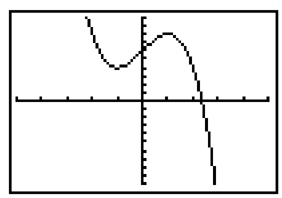
[−10, 10] by [−2, 2]

43. $x \approx 1.77$



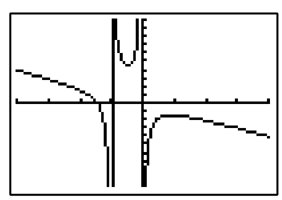
[–5, 5] by [–10, 10]

44. $x \approx 2.36$



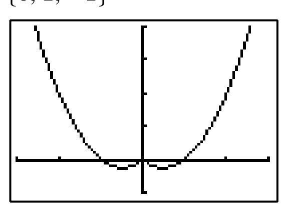
[–5, 5] by [–10, 10]

45. $x \approx -1.47$



[−4, 4] by [−10, 10]

46. $\{0, 1, -1\}$



[−3, 3] by [−1, 4]

47. Model the situation using $C = 0.18x + 32$, where x is the number of miles driven and C is the cost of a day's rental.

(a) Elaine's cost is $0.18(83) + 32 = \$46.94$.

(b) If for Ramon $C = \$69.80$, then

$$x = \frac{69.80 - 32}{0.18} = 210 \text{ miles.}$$

48. **(a)** $4x + 5 - (x^3 + 2x^2 - x + 3) = 0$ or
$-x^3 - 2x^2 + 5x + 2 = 0$

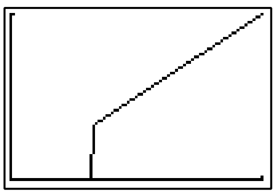
(b) $-x^3 - 2x^2 + 5x + 2 = 0$

(c) A vertical line through the x-intercept of y_3 passes through the point of intersection of y_1 and y_2.

(d) At $x = 1.6813306$, $y_1 = y_2 = 11.725322$.
At $x = -0.3579264$, $y_1 = y_2 = 3.5682944$.
At $x = -3.323404$, $y_1 = y_2 = -8.293616$.

49. **(a)** $y = (x^{200})^{1/200} = x^{200/200} = x^1 = x$ for all $x \geq 0$.

(b) The graph looks like this:



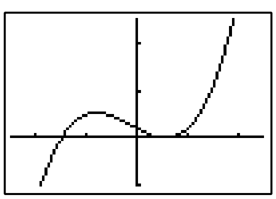
[0, 1] by [0, 1]

(c) Yes, this is different from the graph of $y = x$.

(d) For values of x close to 0, x^{200} is so small that the calculator is unable to distinguish it from zero. It returns a value of $0^{1/200} = 0$ rather than x.

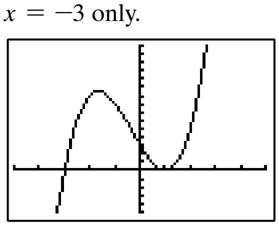
50. The length of each side of the square is $x + b$, so the area of the whole square is $(x + b)^2$. The square is made up of one square with area $x \cdot x = x^2$, one square with area $b \cdot b = b^2$, and two rectangles, each with area $b \cdot x = bx$. Using these four figures, the area of the square is $x^2 + 2bx + b^2$.

51. **(a)** $x = -3$ or $x = 1.1$ or $x = 1.15$.



[−5, 5] by [−200, 500]

(b) $x = -3$ only.



[−10, 10] by [−5, 5]

52. **(a)** Area: $x^2 + x\left(\frac{b}{2}\right) + x\left(\frac{b}{2}\right) = x^2 + bx$

(b) $\frac{b}{2} \cdot \frac{b}{2} = \left(\frac{b}{2}\right)^2$

(c) $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$ is the algebraic formula for completing the square, just as the area $\left(\frac{b}{2}\right)^2$ completes the area $x^2 + bx$ to form the area $\left(x + \frac{b}{2}\right)^2$.

53. Let n be any integer.
$n^2 + 2n = n(n + 2)$, which is either the product of two odd integers or the product of two even integers.

The product of two odd integers is odd.

The product of two even integers is a multiple of 4, since each even integer in the product contributes a factor of 2 to the product.

Therefore, $n^2 + 2n$ is either odd or a multiple of 4.

54. One possible story: The jogger travels at an approximately constant speed throughout her workout. She jogs to the far end of the course, turns around and returns to her starting point, then goes out again for a second trip.

55. False. A product is zero if *any* factor is zero. That is, it takes only one zero factor to make the product zero.

56. False. Predictions are always fallible, and in particular an algebraic model that fits the data well for a certain range of input values may not work for other input values.

57. This is a line with a negative slope and a y-intercept of 12. The answer is C. (The graph checks.)

58. This is the graph of a square root function, but flipped left-over-right. The answer is E. (The graph checks.)

59. As x increases by ones, the y-values get farther and farther apart, which implies an increasing slope and suggests a quadratic equation. The answer is B. (The equation checks.)

60. As x increases by 2's, y increases by 4's, which implies a constant slope of 2. The answer is A. (The equation checks.)

61. **(a)** March

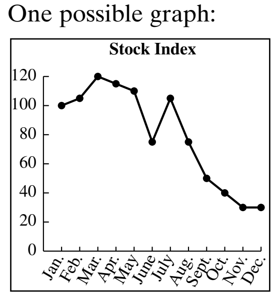
(b) \$120

(c) June, after three months of poor performance

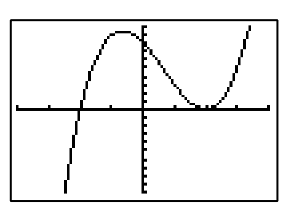
(d) Ahmad paid (100)(\$120) = \$12,000 for the stock and sold it for (100)(\$100) = \$10,000. He lost \$2,000 on the stock.

(e) After reaching a low in June, the stock climbed back to a price near \$140 by December. LaToya's shares had gained \$2000 by that point.

(f) One possible graph:



62. **(a)**



[−4, 4] by [−10, 10]

(b) Factoring, we find $y = (x + 2)(x - 2)(x - 2)$. There is a double zero at $x = 2$, a zero at $x = -2$, and no other zeros (since it is a cubic).

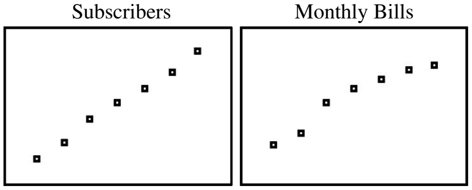
(c) Same visually as the graph in (a).

(d) $b^2 - 4ac$ is the discriminant. In this case, $b^2 - 4ac = (-4)^2 - 4(1)(4.01) = -0.04$, which is negative. So the only real zero of the product $y = (x + 2)(x^2 - 4x + 4.01)$ is at $x = -2$.

(e) Same visually as the graph in (a).

(f) $b^2 - 4ac = (-4)^2 - 4(1)(3.99) = 0.04$, which is positive. The discriminant will provide two real zeros of the quadratic, and $(x + 2)$ provides the third. A cubic equation can have no more than three real roots.

63. (a)



[7, 15] by [50, 200]   [7, 15] by [35, 55]

(b) The graph for subscribers appears to be linear. Since time $t =$ the number of years after 1990, $t = 8$ for 1998 and $t = 14$ for 2004. The slope of the line is

$$\frac{180.4 - 69.2}{14 - 8} = \frac{111.2}{6} \approx 18.53.$$

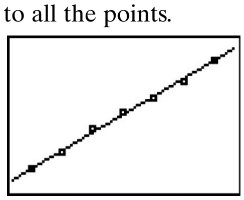
Use the point-slope form to write the equation:
$y - 69.2 = 18.53(x - 8)$.

Solve for y: $\quad y - 69.2 = 18.53x - 148.24$

$$y = 18.53x - 79.04$$

The linear model for subscribers as a function of years is $y = 18.53x - 79.04$.

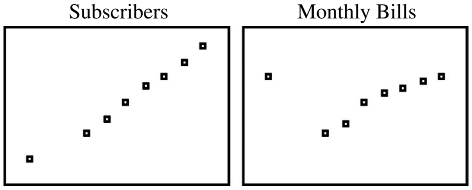
(c) The fit is very good. The line goes through or is close to all the points.



[7, 15] by [50, 200]

(d) The monthly bill scatter plot has a curved shape that could be modeled more effectively by a function with a curved graph. Some possibilities include a quadratic function (parabola), a logarithmic function, a power function (e.g., square root), a logistic function, or a sine function.

(e)



[4, 15] by [10, 200]   [4, 15] by [30, 60]

(f) In 1995, cellular phone technology was still emerging, so the growth rate was not as fast as it was in more recent years. Thus, the slope from 1995 ($t = 5$) to 1998 ($t = 8$) is lower than the slope from 1998 to 2004. Cellular technology was more expensive before competition brought prices down. This explains the anomaly on the monthly bill scatter plot.

64. One possible answer: The number of cell phone users is increasing steadily (as the linear model shows), and the average monthly bill is climbing more slowly as more people share the industry cost. The model shows that the number of users will continue to rise, although the linear model cannot hold up indefinitely.