Section 1.2 Functions and Their Properties

Exploration 1

1. From left to right, the tables are (c) constant, (b) decreasing, and (a) increasing.

2.

X moves from	ΔX	ΔY1	X moves from	ΔX	ΔY2	X moves from	ΔX	ΔY3
−2 to −1	1	0	−2 to −1	1	−2	−2 to −1	1	2
−1 to 0	1	0	−1 to 0	1	−1	−1 to 0	1	2
0 to 1	1	0	0 to 1	1	−2	0 to 1	1	2
1 to 3	2	0	1 to 3	2	−4	1 to 3	2	3
3 to 7	4	0	3 to 7	4	−6	3 to 7	4	6

3. For an increasing function, ΔY/ΔX is positive. For a decreasing function, ΔY/ΔX is negative. For a constant function, ΔY/ΔX is 0.

4. For lines, ΔY/ΔX is the slope. Lines with positive slope are increasing, lines with negative slope are decreasing, and lines with 0 slope are constant, so this supports our answers to part 3.

Quick Review 1.2

1. $x^2 - 16 = 0$
   $x^2 = 16$
   $x = \pm 4$

2. $9 - x^2 = 0$
   $9 = x^2$
   $\pm 3 = x$

3. $x - 10 < 0$
   $x < 10$

4. $5 - x \le 0$
   $-x \le -5$
   $x \ge 5$

5. As we have seen, the denominator of a function cannot be zero.
   We need $x - 16 = 0$
   $x = 16$

6. We need $x^2 - 16 = 0$
   $x^2 = 16$
   $x = \pm 4$

7. We need $x - 16 < 0$
   $x < 16$

8. We need
$$\begin{aligned} x^2 - 1 &= 0 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

9. We need $3 - x \le 0$ and $x + 2 < 0$
$$\begin{aligned} 3 &\le x \qquad & x &< -2 \\ x &< -2 \quad \text{and} & x &\ge 3 \end{aligned}$$

10. We need
$$\begin{aligned} x^2 - 4 &= 0 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

Section 1.2 Exercises

1. Yes, $y = \sqrt{x - 4}$ is a function of x, because when a number is substituted for x, there is at most one value produced for $\sqrt{x - 4}$.

2. No, $y = x^2 \pm 3$ is not a function of x, because when a number is substituted for x, y can be either 3 more or 3 less than x^2.

3. No, $x = 2y^2$ does not determine y as a function of x, because when a positive number is substituted for x, y can be either $\sqrt{\frac{x}{2}}$ or $-\sqrt{\frac{x}{2}}$.

4. Yes, $x = 12 - y$ determines y as a function of x, because when a number is substituted for x, there is exactly one number y which, when subtracted from 12, produces x.

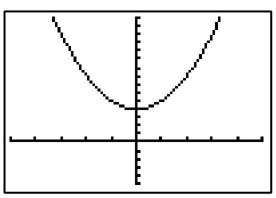
5. Yes

6. No

7. No

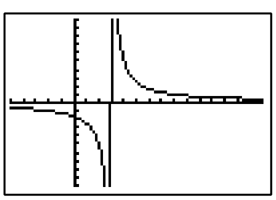
8. Yes

9. We need $x^2 + 4 \ge 0$; this is true for all real x. Domain: $(-\infty, \infty)$.



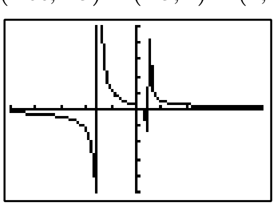
$[-5, 5]$ by $[-5, 15]$

10. We need $x - 3 \ne 0$. Domain: $(-\infty, 3) \cup (3, \infty)$.



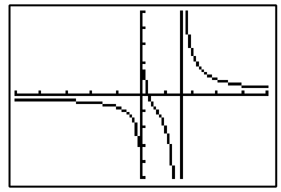
$[-5, 15]$ by $[-10, 10]$

11. We need $x + 3 \ne 0$ and $x - 1 \ne 0$. Domain: $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$.



$[-10, 10]$ by $[-10, 10]$

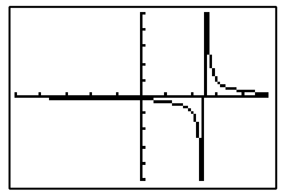
12. We need $x \ne 0$ and $x - 3 \ne 0$. Domain: $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$.



$[-10, 10]$ by $[-10, 10]$

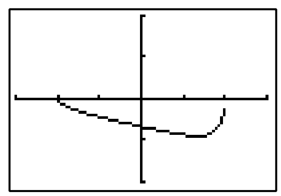
13. We notice that $g(x) = \dfrac{x}{x^2 - 5x} = \dfrac{x}{x(x - 5)}$.

As a result, $x - 5 \ne 0$ and $x \ne 0$.
Domain: $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$.



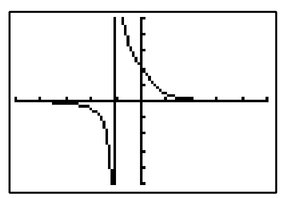
$[-10, 10]$ by $[-5, 5]$

14. We need $x - 3 \ne 0$ and $4 - x^2 \ge 0$. This means $x \ne 3$ and $x^2 \le 4$; the latter implies that $-2 \le x \le 2$, so the domain is $[-2, 2]$.



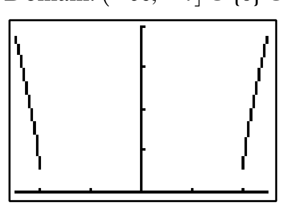
$[-3, 3]$ by $[-2, 2]$

15. We need $x + 1 \ne 0$, $x^2 + 1 \ne 0$, and $4 - x \ge 0$. The first requirement means $x \ne -1$, the second is true for all x, and the last means $x \le 4$. The domain is therefore $(-\infty, -1) \cup (-1, 4]$.



$[-5, 5]$ by $[-5, 5]$

16. We need
$$\begin{aligned} x^4 - 16x^2 &\ge 0 \\ x^2(x^2 - 16) &\ge 0 \\ x^2 = 0 \quad \text{or} \quad x^2 - 16 &\ge 0 \\ x^2 &\ge 16 \end{aligned}$$
$$x = 0 \quad \text{or} \quad x \ge 4, \quad x \le -4$$
Domain: $(-\infty, -4] \cup \{0\} \cup [4, \infty)$

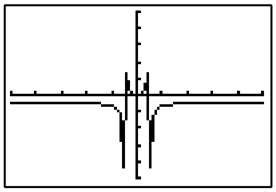


$[-5, 5]$ by $[0, 16]$

17. $f(x) = 10 - x^2$ can take on any negative value. Because x^2 is nonnegative, $f(x)$ cannot be greater than 10. The range is $(-\infty, 10]$.

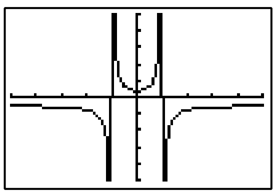
18. $g(x) = 5 + \sqrt{4 - x}$ can take on any value ≥ 5, but because $\sqrt{4 - x}$ is nonnegative, $g(x)$ cannot be less than 5. The range is $[5, \infty)$.

19. The range of a function is most simply found by graphing it. As our graph shows, the range of $f(x)$ is $(-\infty, -1) \cup [0, \infty)$.



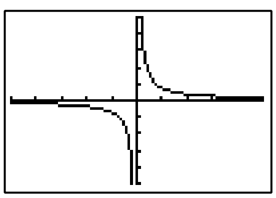
$[-10, 10]$ by $[-10, 10]$

20. As our graph illustrates, the range of $g(x)$ is $(-\infty, -1) \cup [0.75, \infty)$.



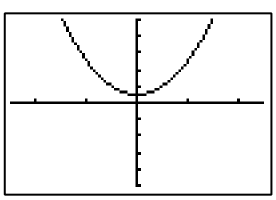
$[-10, 10]$ by $[-10, 10]$

21. Yes, non-removable



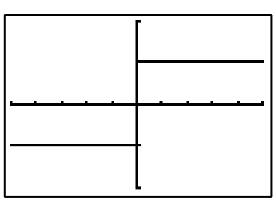
$[-10, 10]$ by $[-10, 10]$

22. Yes, removable



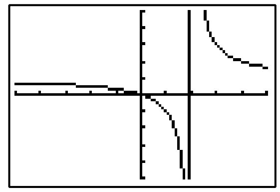
$[-5, 5]$ by $[-10, 10]$

23. Yes, non-removable



$[-10, 10]$ by $[-2, 2]$

24. Yes, non-removable



$[-5, 5]$ by $[-5, 5]$

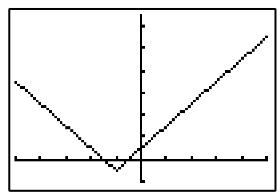
25. Local maxima at $(-1, 4)$ and $(5, 5)$, local minimum at $(2, 2)$. The function increases on $(-\infty, -1]$, decreases on $[-1, 2]$, increases on $[2, 5]$, and decreases on $[5, \infty)$.

26. Local minimum at $(1, 2)$, $(3, 3)$ is neither, and $(5, 7)$ is a local maximum. The function decreases on $(-\infty, 1]$, increases on $[1, 5]$, and decreases on $[5, \infty)$.

27. $(-1, 3)$ and $(3, 3)$ are neither. $(1, 5)$ is a local maximum, and $(5, 1)$ is a local minimum. The function increases on $(-\infty, 1]$, decreases on $[1, 5]$, and increases on $[5, \infty)$.

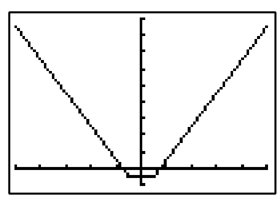
28. $(-1, 1)$ and $(3, 1)$ are local minima, while $(1, 6)$ and $(5, 4)$ are local maxima. The function decreases on $(-\infty, -1]$, increases on $[-1, 1]$, decreases on $(1, 3]$, increases on $[3, 5]$, and decreases on $[5, \infty)$.

29. Decreasing on $(-\infty, -2]$; increasing on $[-2, \infty)$



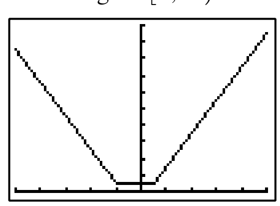
$[-10, 10]$ by $[-2, 18]$

30. Decreasing on $(-\infty, -1]$; constant on $[-1, 1]$; increasing on $[1, \infty)$



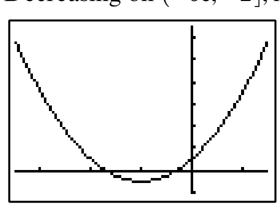
$[-10, 10]$ by $[-2, 18]$

31. Decreasing on $(-\infty, -2]$; constant on $[-2, 1]$; increasing on $[1, \infty)$



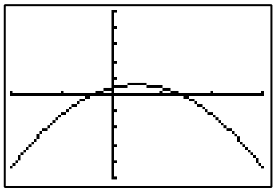
$[-10, 10]$ by $[0, 20]$

32. Decreasing on $(-\infty, -2]$; increasing on $[-2, \infty)$



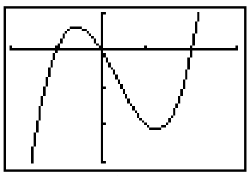
$[-7, 3]$ by $[-2, 13]$

33. Increasing on $(-\infty, 1]$; decreasing on $[1, \infty)$



$[-4, 6]$ by $[-25, 25]$

34. Increasing on $(-\infty, -0.5]$; decreasing on $[-0.5, 1.2]$, increasing on $[1.2, \infty)$. The middle values are approximate —they are actually at about -0.549 and 1.215. The values given are what might be observed on the decimal window.



$[-2, 3]$ by $[-3, 1]$

35. Constant functions are always bounded.

36.
$$x^2 > 0$$
$$-x^2 < 0$$
$$2 - x^2 < 2$$
y is bounded above by $y = 2$.

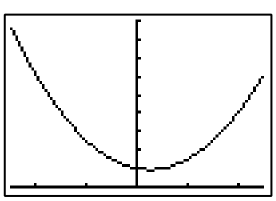
37. $2^x > 0$ for all x, so y is bounded below by $y = 0$.

38. $2^{-x} = \frac{1}{2^x} > 0$ for all x, so y is bounded below by $y = 0$.

39. Since $y = \sqrt{1 - x^2}$ is always positive, we know that $y \geq 0$ for all x. We must also check for an upper bound:
$$x^2 > 0$$
$$-x^2 < 0$$
$$1 - x^2 < 1$$
$$\sqrt{1 - x^2} < \sqrt{1}$$
$$\sqrt{1 - x^2} < 1$$
Thus, y is bounded.

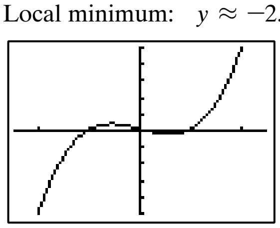
40. There are no restrictions on either x or x^3, so y is not bounded above or below.

41. f has a local minimum when $x = 0.5$, where $y = 3.75$. It has no maximum.



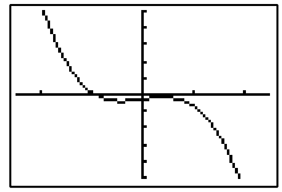
$[-5, 5]$ by $[0, 36]$

42. Local maximum: $y \approx 4.08$ at $x \approx -1.15$.
Local minimum: $y \approx -2.08$ at $x \approx 1.15$.



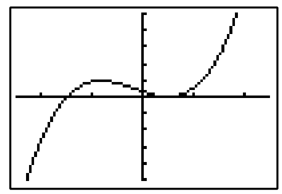
$[-5, 5]$ by $[-50, 50]$

43. Local minimum: $y \approx -4.09$ at $x \approx -0.82$.
Local maximum: $y \approx -1.91$ at $x \approx 0.82$.



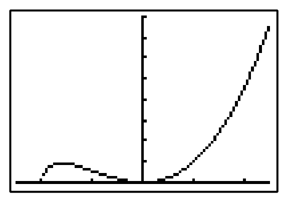
$[-5, 5]$ by $[-50, 50]$

44. Local maximum: $y \approx 9.48$ at $x \approx -1.67$.
Local minimum: $y = 0$ when $x = 1$.



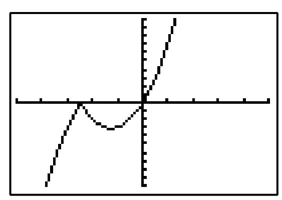
$[-5, 5]$ by $[-50, 50]$

45. Local maximum: $y \approx 9.16$ at $x \approx -3.20$.
Local minima: $y = 0$ at $x = 0$ and $y = 0$ at $x = -4$.



$[-5, 5]$ by $[0, 80]$

46. Local maximum: $y = 0$ at $x = -2.5$.
Local minimum: $y \approx -3.13$ at $x = -1.25$.



$[-5, 5]$ by $[-10, 10]$

47. Even: $f(-x) = 2(-x)^4 = 2x^4 = f(x)$

48. Odd: $g(-x) = (-x)^3 = -x^3 = -g(x)$

49. Even: $f(-x) = \sqrt{(-x)^2 + 2} = \sqrt{x^2 + 2} = f(x)$

50. Even: $g(-x) = \dfrac{3}{1 + (-x)^2} = \dfrac{3}{1 + x^2} = g(x)$

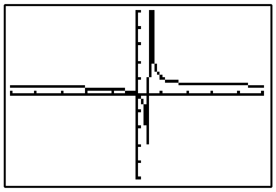
51. Neither: $f(-x) = -(-x)^2 + 0.03(-x) + 5 = -x^2 - 0.03x + 5$, which is neither $f(x)$ nor $-f(x)$.

52. Neither: $f(-x) = (-x)^3 + 0.04(-x)^2 + 3 = -x^3 + 0.04x^2 + 3$, which is neither $f(x)$ nor $-f(x)$.

53. Odd: $g(-x) = 2(-x)^3 - 3(-x)$
$= -2x^3 + 3x = -g(x)$

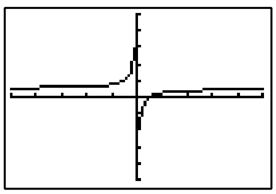
54. Odd: $h(-x) = \dfrac{1}{-x} = -\dfrac{1}{x} = -h(x)$

55. The quotient $\frac{x}{x-1}$ is undefined at $x = 1$, indicating that $x = 1$ is a vertical asymptote. Similarly, $\lim_{x\to\infty} \frac{x}{x-1} = 1$, indicating a horizontal asymptote at $y = 1$. The graph confirms these.



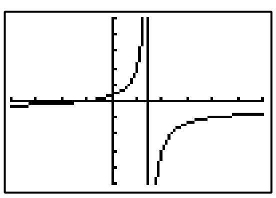
$[-10, 10]$ by $[-10, 10]$

56. The quotient $\frac{x-1}{x}$ is undefined at $x = 0$, indicating a possible vertical asymptote at $x = 0$. Similarly, $\lim_{x\to\infty} \frac{x-1}{x} = 1$, indicating a possible horizontal asymptote at $y = 1$. The graph confirms those asymptotes.



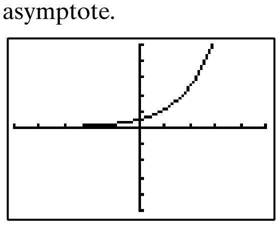
$[-10, 10]$ by $[-10, 10]$

57. The quotient $\frac{x+2}{3-x}$ is undefined at $x = 3$, indicating a possible vertical asymptote at $x = 3$. Similarly, $\lim_{x\to\infty} \frac{x+2}{3-x} = -1$, indicating a possible horizontal asymptote at $y = -1$. The graph confirms these asymptotes.



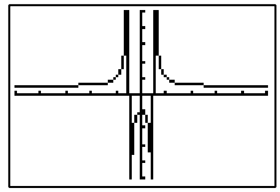
$[-8, 12]$ by $[-10, 10]$

58. Since $g(x)$ is continuous over $-\infty < x < \infty$, we do not expect a vertical asymptote. However, $\lim_{x\to-\infty} 1.5^x = \lim_{x\to\infty} 1.5^{-x} = \lim_{x\to\infty} \frac{1}{1.5^x} = 0$, so we expect a horizontal asymptote $y = 0$. The graph confirms this asymptote.



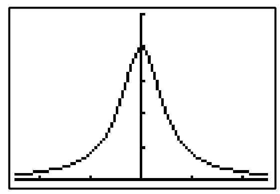
$[-10, 10]$ by $[-10, 10]$

59. The quotient $\frac{x^2+2}{x^2-1}$ is undefined at $x = 1$ and $x = -1$. So we expect two vertical asymptotes. Similarly, the $\lim_{x\to\infty} \frac{x^2+2}{x^2-1} = 1$, so we expect a horizontal asymptote at $y = 1$. The graph confirms these asymptotes.



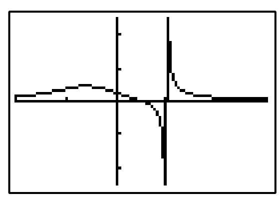
$[-10, 10]$ by $[-10, 10]$

60. We note that $x^2 + 1 > 0$ for $-\infty < x < \infty$, so we do not expect a vertical asymptote. However, $\lim_{x\to\infty} \frac{4}{x^2+1} = 0$, so we expect a horizontal asymptote at $y = 0$. The graph confirms this.



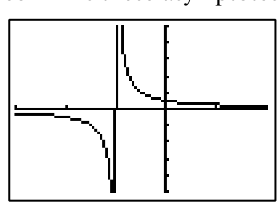
$[-5, 5]$ by $[0, 5]$

61. The quotient $\frac{4x-4}{x^3-8}$ does not exist at $x = 2$, so we expect a vertical asymptote there. Similarly, $\lim_{x\to\infty} \frac{4x-4}{x^3+8} = 0$, so we expect a horizontal asymptote at $y = 0$. The graph confirms these asymptotes.



$[-4, 6]$ by $[-5, 5]$

62. The quotient $\frac{2x-4}{x^2-4} = \frac{2(x-2)}{(x-2)(x+2)} = \frac{2}{x+2}$. Since $x = 2$ is a removable discontinuity, we expect a vertical asymptote at only $x = -2$. Similarly, $\lim_{x\to\infty} \frac{2}{x-2} = 0$, so we expect a horizontal asymptote at $y = 0$. The graph confirms these asymptotes.



$[-6, 4]$ by $[-10, 10]$

63. The denominator is zero when $x = -\frac{1}{2}$, so there is a vertical asymptote at $x = -\frac{1}{2}$. When x is very large, $\frac{x+2}{2x+1}$ behaves much like $\frac{x}{2x} = \frac{1}{2}$, so there is a horizontal asymptote at $y = \frac{1}{2}$. The graph matching this description is (b).

64. The denominator is zero when $x = -\frac{1}{2}$, so there is a vertical asymptote at $x = -\frac{1}{2}$. When x is very large, $\frac{x^2+2}{2x+1}$ behaves much like $\frac{x^2}{2x} = \frac{x}{2}$, so $y = \frac{x}{2}$ is a slant asymptote. The graph matching this description is (c).

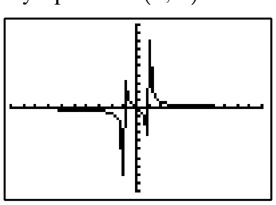
65. The denominator cannot equal zero, so there is no vertical asymptote. When x is very large, $\frac{x+2}{2x^2+1}$ behaves much like $\frac{x}{2x^2} = \frac{1}{2x}$, which for large x is close to zero. So there is a horizontal asymptote at $y = 0$. The graph matching this description is (a).

66. The denominator cannot equal zero, so there is no vertical asymptote. When x is very large, $\frac{x^3+2}{2x^2+1}$ behaves much like $\frac{x^3}{2x^2} = \frac{x}{2}$, so $y = \frac{x}{2}$ is a slant asymptote. The graph matching this description is (d).

67. (a) Since, $\lim_{x\to\infty} \frac{x}{x^2-1} = 0$, we expect a horizontal asymptote at $y = 0$. To find where our function crosses $y = 0$, we solve the equation

$$\frac{x}{x^2-1} = 0$$
$$x = 0 \cdot (x^2 - 1)$$
$$x = 0$$

The graph confirms that $f(x)$ crosses the horizontal asymptote at $(0, 0)$.

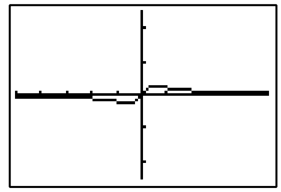


[−10, 10] by [−10, 10]

(b) Since $\lim_{x\to\infty} \frac{x}{x^2+1} = 0$, we expect a horizontal asymptote at $y = 0$. To find where our function crosses $y = 0$, we solve the equation:

$$\frac{x}{x^2+1} = 0$$
$$x = 0 \cdot (x^2 + 1)$$
$$x = 0$$

The graph confirms that $g(x)$ crosses the horizontal asymptote at $(0, 0)$.

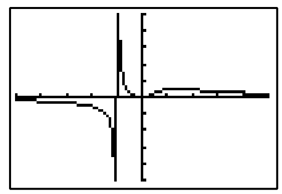


[−10, 10] by [−5, 5]

(c) Since $\lim_{x\to\infty} \frac{x^2}{x^3+1} = 0$, we expect a horizontal asymptote at $y = 0$. To find where $h(x)$ crosses $y = 0$, we solve the equation

$$\frac{x^2}{x^3+1} = 0$$
$$x^2 = 0 \cdot (x^3 + 1)$$
$$x^2 = 0$$
$$x = 0$$

The graph confirms that $h(x)$ intersects the horizontal asymptote at $(0, 0)$.



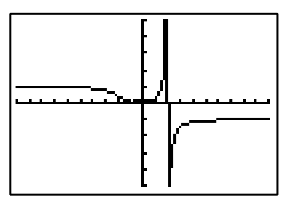
[−5, 5] by [−5, 5]

68. We find **(a)** and **(c)** have graphs with more than one horizontal asymptote as follows:

(a) To find horizontal asymptotes, we check limits, at $x \to \infty$ and $x \to -\infty$. We also know that our numerator $|x^3 + 1|$, is positive for all x, and that our denominator, $8 - x^3$, is positive for $x < 2$ and negative for $x > 2$. Considering these two statements, we find

$$\lim_{x\to\infty} \frac{|x^3+1|}{8-x^3} = -1 \text{ and } \lim_{x\to-\infty} \frac{|x^3+1|}{8-x^3} = 1.$$

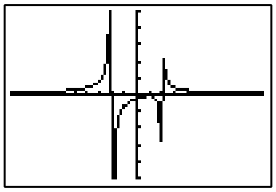
The graph confirms that we have horizontal asymptotes at $y = 1$ and $y = -1$.



[−10, 10] by [−5, 5]

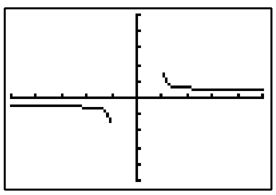
(b) Again, we see that our numerator, $|x - 1|$, is positive for all x. As a result, $g(x)$ can be negative only when $x^2 - 4 < 0$, and $g(x)$ can be positive only when $x^2 - 4 > 0$. This means that $g(x)$ can be negative only when $-2 < x < 2$; if $x < -2$ or $x > 2$, $g(x)$ will be positive. As a result, we know that

$$\lim_{x\to\infty} \frac{|x-1|}{x^2-4} = \lim_{x\to-\infty} \frac{|x-1|}{x^2-4} = 0,$$ giving just one horizontal asymptote at $y = 0$. Our graph confirms this asymptote.



$[-5, 5]$ by $[-5, 15]$

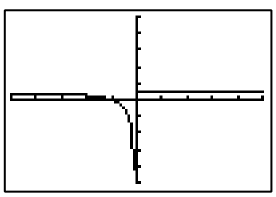
(c) As we demonstrated earlier, we need $x^2 - 4 > 0$ otherwise our function is not defined within the real numbers. As a result, we know that our denominator, $\sqrt{x^2 - 4}$, is always positive [and that $h(x)$ is defined only in the domain $(-\infty, -2) \cup (2, \infty)$]. Checking limits, we find $\lim_{x\to\infty} \frac{x}{\sqrt{x^2 - 4}} = 1$ and $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2 - 4}} = -1$. The graph confirms that we have horizontal asymptotes at $y = 1$ and $y = -1$.



$[-10, 10]$ by $[-10, 10]$

69. (a) The vertical asymptote is $x = 0$, and this function is undefined at $x = 0$ (because a denominator can't be zero).

(b)



$[-10, 10]$ by $[-10, 10]$

Add the point $(0, 0)$.

(c) Yes. It passes the vertical line test.

70. The horizontal asymptotes are determined by the two limits, $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to+\infty} f(x)$. These are at most two different numbers.

71. True. This is what it means for a set of points to be the graph of a function.

72. False. There are many function graphs that are symmetric with respect to the x-axis. One example is $f(x) = 0$.

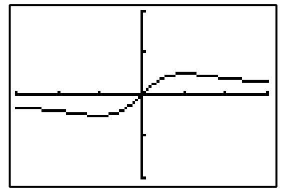
73. Temperature is a continuous variable, whereas the other quantities all vary in steps. The answer is B.

74. "Number of balls" represents a whole number, so that the quantity changes in jumps as the ball radius is altered. The answer is C.

75. Air pressure drops with increasing height. All the other functions either steadily increase or else go both up and down. The answer is C.

76. The height of a swinging pendulum goes up and down over time as the pendulum swings back and forth. The answer is E.

77. (a)



$[-3, 3]$ by $[-2, 2]$

$k = 1$

(b) $\frac{x}{1 + x^2} < 1 \Leftrightarrow x < 1 + x^2 \Leftrightarrow x^2 - x + 1 > 0$

But the discriminant of $x^2 - x + 1$ is negative (-3), so the graph never crosses the x-axis on the interval $(0, \infty)$.

(c) $k = -1$

(d) $\frac{x}{1 + x^2} > -1 \Leftrightarrow x > -1 - x^2 \Leftrightarrow x^2 + x + 1 > 0$

But the discriminant of $x^2 + x + 1$ is negative (-3), so the graph never crosses the x-axis on the interval $(-\infty, 0)$.

78. (a) Increasing

(b)

Δy
1
1.05
0.52
0.43
0.36
0.33
0.31
0.28

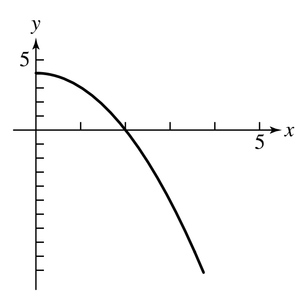
(c)

$\Delta\Delta y$
0.05
−0.53
−0.09
−0.07
−0.03
−0.02
−0.03

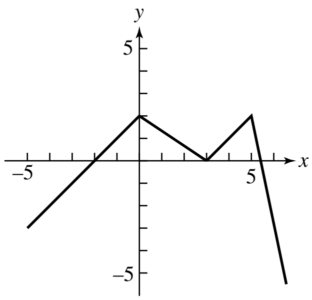
Δy is none of these since it first increases from 1 to 1.05 and then decreases.

(d) The graph rises, but bends downward as it rises.

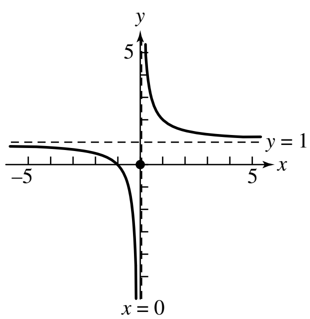
(e) An example:



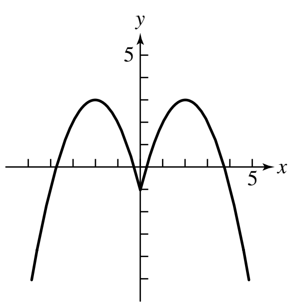
79. One possible graph:



80. One possible graph:



81. One possible graph:



82. Answers vary.

83. (a)

$$x^2 > 0$$
$$-0.8x^2 < 0$$
$$2 - 0.8x^2 < 2$$

$f(x)$ is bounded above by $y = 2$. To determine if $y = 2$ is in the range, we must solve the equation for x: $2 = 2 - 0.8x^2$

$0 = -0.8x^2$

$0 = x^2$

$0 = x$

Since $f(x)$ exists at $x = 0$, $y = 2$ is in the range.

(b) $\lim_{x\to\infty} \frac{3x^2}{3 + x^2} = \lim_{x\to\infty} \frac{3x^2}{x^2} = \lim_{x\to\infty} 3 = 3$. Thus, $g(x)$ is bounded by $y = 3$. However, when we solve for x, we get $3 = \frac{3x^2}{3 + x^2}$

$$3(3 + x^2) = 3x^2$$
$$9 + 3x^2 = 3x^2$$
$$9 = 0$$

Since $9 \neq 0$, $y = 3$ is not in the range of $g(x)$.

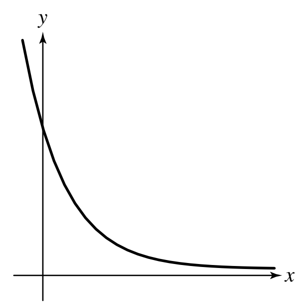
(c) $h(x)$ is not bounded above.

(d) For all values of x, we know that $\sin(x)$ is bounded above by $y = 1$. Similarly, $2\sin(x)$ is bounded above by $y = 2 \cdot 1 = 2$. It is in the range.

(e) $\lim_{x\to\infty} \frac{4x}{x^2 + 2x + 1} = \lim_{x\to\infty} \frac{4x}{(x + 1)^2} =$
$\lim_{x\to\infty} 4\left(\frac{x}{x + 1}\right)\left(\frac{1}{x + 1}\right) = \lim_{x\to\infty} \frac{4}{x + 1}$
(since $x + 1 \approx x$ for very large x) $= 0$.

[Similarly, $\lim_{x\to-\infty} \frac{4x}{x^2 + x + 1} = 0$] As a result, we know that $g(x)$ is bounded by $y = 0$ as x goes to ∞ and $-\infty$.

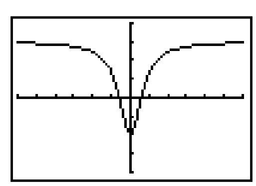
However, $g(x) > 0$ for all $x > 0$ (since $(x + 1)^2 > 0$ always and $4x > 0$ when $x > 0$), so we must check points near $x = 0$ to determine where the function is at its maximum. [Since $g(x) < 0$ for all $x < 0$ (since $(x + 1)^2 > 0$ always and $4x < 0$ when $x < 0$) we can ignore those values of x since we are concerned only with the upper bound of $g(x)$.] Examining our graph, we see that $g(x)$ has an upper bound at $y = 1$, which occurs when $x = 1$. The least upper bound of $g(x) = 1$, and it is in the range of $g(x)$.



84. As the graph moves continuously from the point $(-1, 5)$ down to the point $(1, -5)$, it must cross the x-axis somewhere along the way. That x-intercept will be a zero of the function in the interval $[-1, 1]$.

85. Since f is odd, $f(-x) = -f(x)$ for all x. In particular, $f(-0) = -f(0)$. This is equivalent to saying that $f(0) = -f(0)$, and the only number which equals its opposite is 0. Therefore, $f(0) = 0$, which means the graph must pass through the origin.

86.



[–6, 6] by [–2, 2]

(a) $y = 1.5$

(b) $[-1, 1.5]$

(c) $-1 \leq \frac{3x^2 - 1}{2x^2 + 1} \leq 1.5$

$$0 \leq 1 + \frac{3x^2 - 1}{2x^2 + 1} \leq 2.5$$
$$0 \leq 2x^2 + 1 + 3x^2 - 1 \leq 5x^2 + 2.5$$
$$0 \leq 5x^2 \leq 5x^2 + 2.5$$

True for all x.