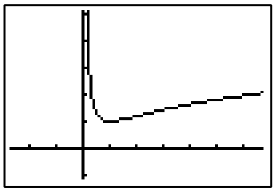
Section 1.3 Twelve Basic Functions

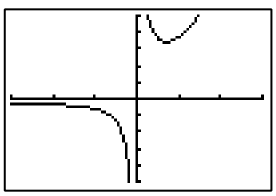
Exploration 1

1. The graphs of $f(x) = \frac{1}{x}$ and $f(x) = \ln x$ have vertical asymptotes at $x = 0$.
2. The graph of $g(x) = \frac{1}{x} + \ln x$ (shown below) does have a vertical asymptote at $x = 0$.



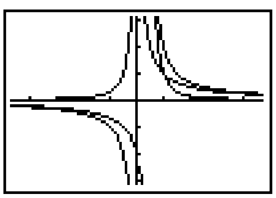
$[-2.7, 6.7]$ by $[-1.1, 5.1]$

3. The graphs of $f(x) = \frac{1}{x}$, $f(x) = e^x$, and $f(x) = \frac{1}{1 + e^{-x}}$ have horizontal asymptotes at $y = 0$.
4. The graph of $g(x) = \frac{1}{x} + e^x$ (shown below) does have a horizontal asymptote at $y = 0$.



$[-3, 3]$ by $[-5, 5]$

5.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

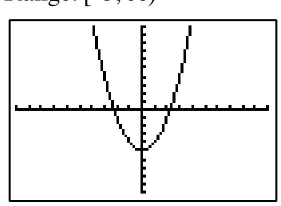
Both $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{2x^2 - x} = \frac{1}{x(2x - 1)}$ have vertical asymptotes at $x = 0$, but $h(x) = f(x) + g(x)$ does not; it has a removable discontinuity.

Quick Review 1.3

1. 59.34
2. $5 - \pi$
3. $7 - \pi$
4. 3
5. 0
6. 1
7. 3
8. -15
9. -4
10. $|1 - \pi| - \pi = (\pi - 1) - \pi = \pi - 1 - \pi = -1$

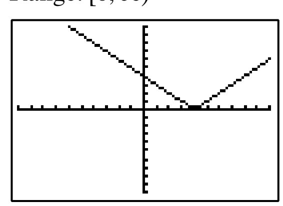
Section 1.3 Exercises

1. $y = x^3 + 1$; (e)
2. $y = |x| - 2$; (g)
3. $y = -\sqrt{x}$; (j)
4. $y = -\sin x$ or $y = \sin(-x)$; (a)
5. $y = -x$; (i)
6. $y = (x - 1)^2$; (f)
7. $y = \text{int}(x + 1)$; (k)
8. $y = -\frac{1}{x}$; (h)
9. $y = (x + 2)^3$; (d)
10. $y = e^x - 2$; (c)
11. $2 - \frac{4}{1 + e^{-x}}$; (l)
12. $y = \cos x + 1$; (b)
13. Exercise 8
14. Exercise 3
15. Exercises 7, 8
16. Exercise 7 (Remember that a continuous function is one that is continuous at every point *in its domain.*)
17. Exercises 2, 4, 6, 10, 11, 12
18. Exercises 3, 4, 11, 12
19. $y = x$, $y = x^3$, $y = \frac{1}{x}$, $y = \sin x$
20. $y = x$, $y = x^3$, $y = \sqrt{x}$, $y = e^x$, $y = \ln x$, $y = \frac{1}{1 + e^{-x}}$
21. $y = x^2$, $y = \frac{1}{x}$, $y = |x|$
22. $y = \sin x$, $y = \cos x$, $y = \text{int}(x)$
23. $y = \frac{1}{x}$, $y = e^x$, $y = \frac{1}{1 + e^{-x}}$
24. $y = x$, $y = x^3$, $y = \ln x$
25. $y = \frac{1}{x}$, $y = \sin x$, $y = \cos x$, $y = \frac{1}{1 + e^{-x}}$
26. $y = x$, $y = x^3$, $y = \text{int}(x)$
27. $y = x$, $y = x^3$, $y = \frac{1}{x}$, $y = \sin x$
28. $y = \sin x$, $y = \cos x$
29. Domain: All reals
Range: $[-5, \infty)$



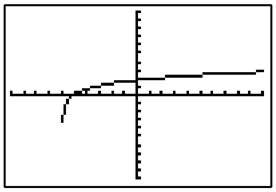
$[-10, 10]$ by $[-10, 10]$

30. Domain: All reals
Range: $[0, \infty)$



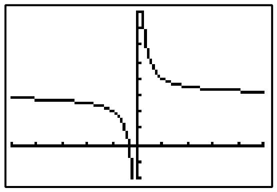
$[-10, 10]$ by $[-10, 10]$

31. Domain: $(-6, \infty)$
Range: All reals



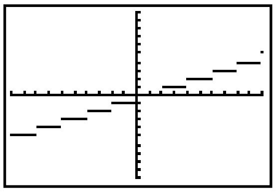
$[-10, 10]$ by $[-10, 10]$

32. Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 3) \cup (3, \infty)$



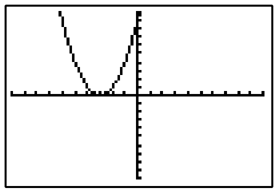
$[-5, 5]$ by $[-2, 8]$

33. Domain: All reals
Range: All integers



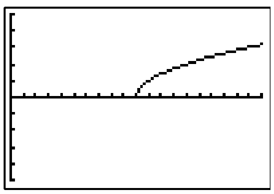
$[-10, 10]$ by $[-10, 10]$

34. Domain: All reals
Range: $[0, \infty)$



$[-10, 10]$ by $[-10, 10]$

35.



$[0, 20]$ by $[-5, 5]$

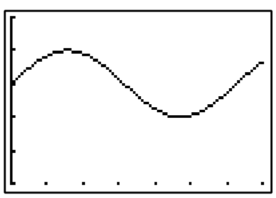
(a) $r(x)$ is increasing on $[10, \infty)$.

(b) $r(x)$ is neither odd nor even.

(c) The one extreme is a minimum value of 0 at $x = 10$.

(d) $r(x) = \sqrt{x - 10}$ is the square root function, shifted 10 units right.

36.



$[0, 7]$ by $[2, 7]$

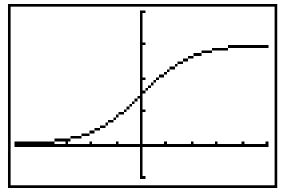
(a) $f(x)$ is increasing on $\left[(2k - 1)\frac{\pi}{2}, (2k + 1)\frac{\pi}{2}\right]$ and decreasing on $\left[(2k + 1)\frac{\pi}{2}, (2k + 3)\frac{\pi}{2}\right]$, where k is an even integer.

(b) $f(x)$ is neither odd nor even.

(c) There are minimum values of 4 at $x = (2k - 1)\frac{\pi}{2}$ and maximum values of 6 at $x = (2k + 1)\frac{\pi}{2}$, where k is an even integer.

(d) $f(x) = \sin(x) + 5$ is the sine function, $\sin x$, shifted 5 units up.

37.



$[-5, 5]$ by $[-1, 4]$

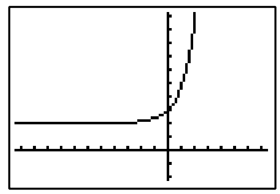
(a) $f(x)$ is increasing on $(-\infty, \infty)$.

(b) $f(x)$ is neither odd nor even.

(c) There are no extrema.

(d) $f(x) = \frac{3}{1 + e^{-x}}$ is the logistic function, $\frac{1}{1 + e^{-x}}$, stretched vertically by a factor of 3.

38.



$[-11.4, 7.4]$ by $[-2.2, 10.2]$

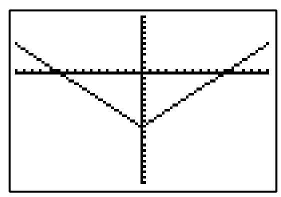
(a) $q(x)$ is increasing on $(-\infty, \infty)$.

(b) $q(x)$ is neither odd nor even.

(c) There are no extrema.

(d) $q(x) = e^x + 2$ is the exponential function, e^x, shifted 2 units up.

39.



$[-15, 15]$ by $[-20, 10]$

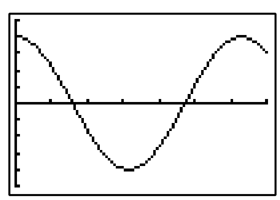
(a) $h(x)$ is increasing on $[0, \infty)$ and decreasing on $(-\infty, 0]$.

(b) $h(x)$ is even, because it is symmetric about the y-axis.

(c) The one extremum is a minimum value of -10 at $x = 0$.

(d) $h(x) = |x| - 10$ is the absolute value function, $|x|$, shifted 10 units down.

40.



$[0, 7]$ by $[-5, 5]$

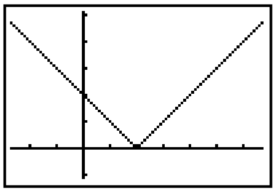
(a) $g(x)$ is increasing on $[(2k - 1)\pi, 2k\pi]$ and decreasing on $[2k\pi, (2k + 1)\pi]$, where k is an integer.

(b) $g(x)$ is even, because it is symmetric about the y-axis.

(c) There are minimum values of -4 at $x = (2k - 1)\pi$ and maximum values of 4 at $x = 2k\pi$, where k is an integer.

(d) $g(x) = 4\cos(x)$ is the cosine function, $\cos x$, stretched vertically by a factor of 4.

41.



[–2.7, 6.7] by [–1.1, 5.1]

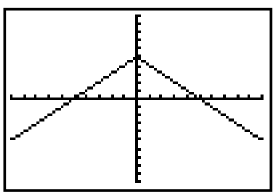
(a) $s(x)$ is increasing on $[2, \infty)$ and decreasing on $(-\infty, 2]$.

(b) $s(x)$ is neither odd nor even.

(c) The one extremum is a minimum value of 0 at $x = 2$.

(d) $s(x) = |x - 2|$ is the absolute value function, $|x|$, shifted 2 units to the right.

42.



[–10, 10] by [–10, 10]

(a) $f(x)$ is increasing on $(-\infty, 0]$ and decreasing on $[0, \infty)$.

(b) $f(x)$ is even, because it is symmetric about the y-axis.

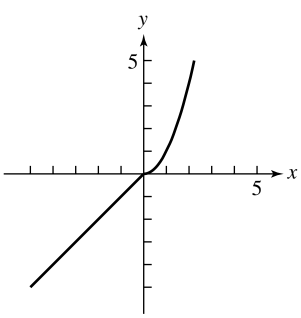
(c) The one extremum is a maximum value of 5 at $x = 0$.

(d) $f(x) = 5 - \text{abs}(x)$ is the absolute value function, $\text{abs}(x)$, reflected across the x-axis and then shifted 5 units up.

43. The end behavior approaches the horizontal asymptotes $y = 2$ and $y = -2$.

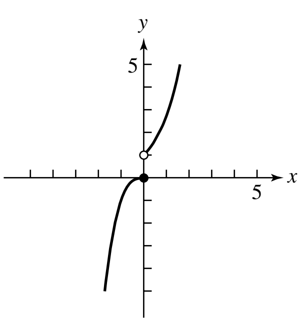
44. The end behavior approaches the horizontal asymptotes $y = 0$ and $y = 3$.

45.



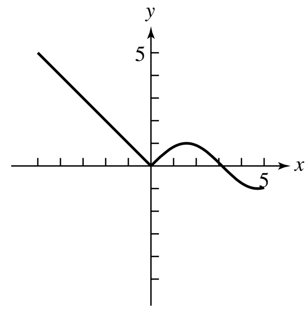
There are no points of discontinuity.

46.



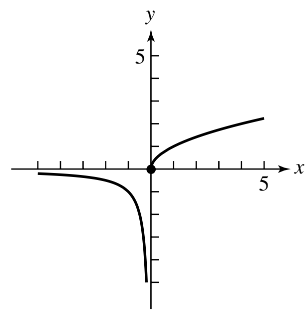
There is a point of discontinuity at $x = 0$.

47.



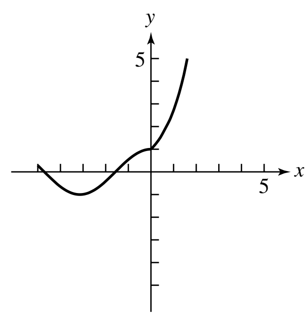
There are no points of discontinuity.

48.



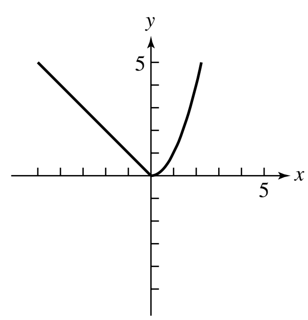
There is a point of discontinuity at $x = 0$.

49.



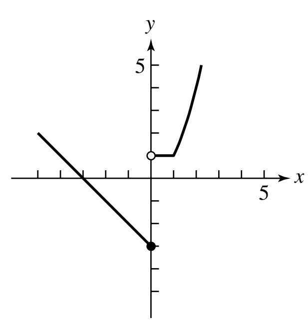
There are no points of discontinuity.

50.



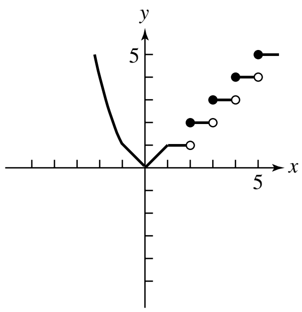
There are no points of discontinuity.

51.



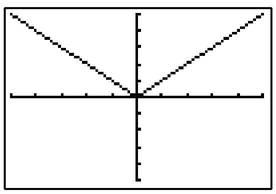
There is a point of discontinuity at $x = 0$.

52.



There are points of discontinuity at $x = 2, 3, 4, 5, \ldots$.

53. (a)

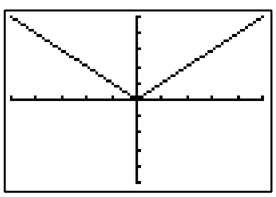


$[-5, 5]$ by $[-5, 5]$

This is $g(x) = |x|$.

(b) Squaring x and taking the (positive) square root has the same effect as the absolute value function.
$f(x) = \sqrt{x^2} = \sqrt{|x|^2} = |x| = g(x)$

54. (a)

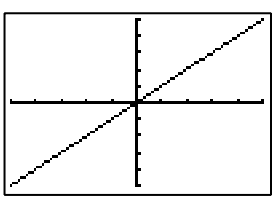


$[-5, 5]$ by $[-5, 5]$

This appears to be $f(x) = |x|$.

(b) For example, $g(1) \approx 0.99 \neq f(1) = 1$.

55. (a)

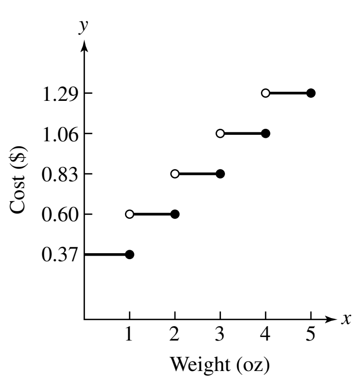


$[-5, 5]$ by $[-5, 5]$

This is the function $f(x) = x$.

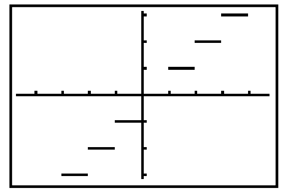
(b) The fact that $\ln(e^x) = x$ shows that the natural logarithm function takes on arbitrarily large values. In particular, it takes on the value L when $x = e^L$.

56. (a)



(b) One possible answer: It is similar because it has discontinuities spaced at regular intervals. It is different because its domain is the set of positive real numbers, and because it is constant on intervals of the form $(k, k + 1]$ instead of $[k, k + 1)$, where k is an integer.

57. The Greatest Integer Function $f(x) = \text{int}(x)$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Domain: all real numbers
Range: all integers
Continuity: There is a discontinuity at each integer value of x.
Increasing/decreasing behavior: constant on intervals of the form $[k, k + 1)$, where k is an integer
Symmetry: none
Boundedness: not bounded
Local extrema: every non-integer is both a local minimum and local maximum
Horizontal asymptotes: none
Vertical asymptotes: none
End behavior: $\text{int}(x) \to -\infty$ as $x \to -\infty$ and $\text{int}(x) \to \infty$ as $x \to \infty$.

58. False. Because the greatest integer function is not one-to-one, its inverse relation is not a function.

59. True. The asymptotes are $x = 0$ and $x = 1$.

60. Because $3 - \dfrac{1}{x} \neq 3$, $0 < \dfrac{5}{1 + e^{-x}} < 5$, $-4 \leq 4 \cos x \leq 4$, and $\text{int}(x - 2)$ takes only integer values. The answer is A.

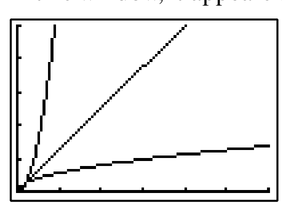
61. $3 < 3 + \dfrac{1}{1 + e^{-x}} < 4$. The answer is D.

62. By comparison of the graphs, the answer is C.

63. The answer is E. The others all have either a restricted domain or intervals where the function is decreasing or constant.

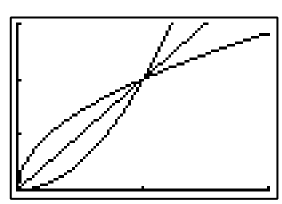
64. (a) Answers will vary.

(b) In this window, it appears that $\sqrt{x} < x < x^2$:



$[0, 30]$ by $[0, 20]$

(c)



$[0, 2]$ by $[0, 1.5]$

(d) On the interval $(0, 1)$, $x^2 < x < \sqrt{x}$.
On the interval $(1, \infty)$, $\sqrt{x} < x < x^2$.
All three functions equal 1 when $x = 1$.

65. (a) A product of two odd functions is even.

(b) A product of two even functions is even.

(c) A product of an odd function and an even function is odd.

66. Answers vary.

67. (a) Pepperoni count ought to be proportional to the area of the pizza, which is proportional to the square of the radius.

(b) $12 = k(4)^2$

$$k = \frac{12}{16} = \frac{3}{4} = 0.75$$

(c) Yes, very well.

(d) The fact that the pepperoni count fits the expected quadratic model so perfectly suggests that the pizzeria uses such a chart. If repeated observations produced the same results, there would be little doubt.

68. (a) $y = e^x$ and $y = \ln x$

(b) $y = x$ and $y = \dfrac{1}{x}$

(c) With domain $[0, \infty)$, $y = x^2$ becomes the inverse of $y = \sqrt{x}$.

69. (a) At $x = 0$, $\dfrac{1}{x}$ does not exist, $e^x = 1$, $\ln x$ is not defined, $\cos x = 1$, and $\dfrac{1}{1 + e^{-x}} = 1$.

(b) for $f(x) = x$, $f(x + y) = x + y = f(x) + f(y)$

(c) for $f(x) = e^x$, $f(xy) = e^{xy} = e^x e^y = f(x) \cdot f(y)$

(d) for $f(x) = \ln x$, $f(x + y) = \ln(xy) = \ln(x) + \ln(y)$
$= f(x) + f(y)$

(e) The odd functions: x, x^3, $\dfrac{1}{x}$, $\sin x$