What you'll learn about

- What Graphs Can Tell Us
- Twelve Basic Functions
- Analyzing Functions Graphically

... and why

As you continue to study mathematics, you will find that the twelve basic functions presented here will come up again and again. By knowing their basic properties, you will recognize them when you see them.

1.3 Twelve Basic Functions

What Graphs Can Tell Us

The preceding section has given us a vocabulary for talking about functions and their properties. We have an entire book ahead of us to study these functions in depth, but in this section we want to set the scene by just *looking* at the graphs of twelve "basic" functions that are available on your graphing calculator.

You will find that function attributes such as domain, range, continuity, asymptotes, extrema, increasingness, decreasingness, and end behavior are every bit as graphical as they are algebraic. Moreover, the visual cues are often much easier to spot than the algebraic ones.

In future chapters you will learn more about the algebraic properties that make these functions behave as they do. Only then will you able to *prove* what is visually apparent in these graphs.
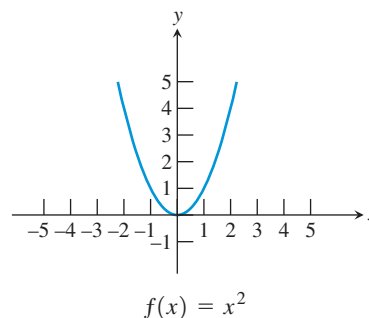
Twelve Basic Functions

The Identity Function

$$f(x) = x$$
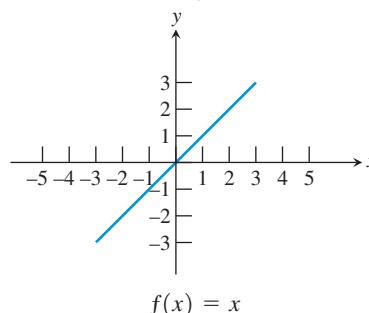
Interesting fact: This is the only function that acts on every real number by leaving it alone.

FIGURE 1.36

The Squaring Function

$$f(x) = x^2$$
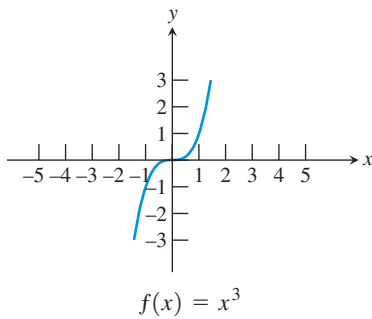
Interesting fact: The graph of this function, called a parabola, has a reflection property that is useful in making flashlights and satellite dishes.
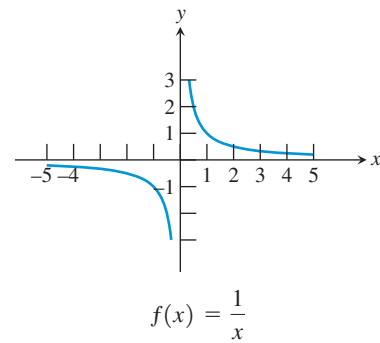
FIGURE 1.37

The Cubing Function

$$f(x) = x^3$$

Interesting fact: The origin is called a "point of inflection" for this curve because the graph changes curvature at that point.
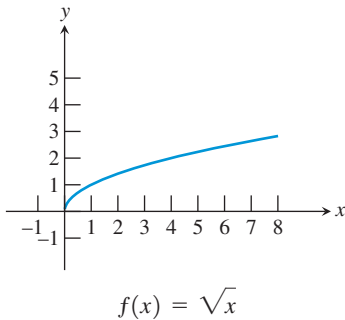
FIGURE 1.38

The Reciprocal Function

$$f(x) = \frac{1}{x}$$

Interesting fact: This curve, called a hyperbola, also has a reflection property that is useful in satellite dishes.

FIGURE 1.39

The Square Root Function
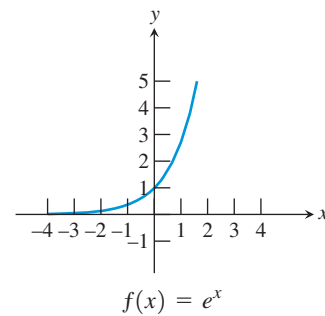

$$f(x) = \sqrt{x}$$

Interesting fact: Put any positive number into your calculator. Take the square root. Then take the square root again. Then take the square root again, and so on. Eventually you will always get 1.
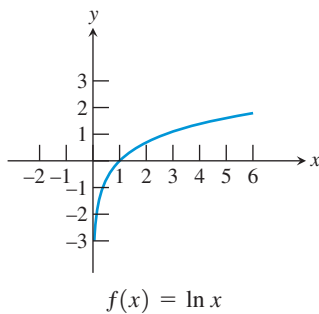
FIGURE 1.40

The Exponential Function

$$f(x) = e^x$$

Interesting fact: The number e is an irrational number (like π) that shows up in a variety of applications. The symbols e and π were both brought into popular use by the great Swiss mathematician Leonhard Euler (1707–1783).

FIGURE 1.41

The Natural Logarithm Function

$$f(x) = \ln x$$

Interesting fact: This function increases very slowly. If the x-axis and y-axis were both scaled with unit lengths of one inch, you would have to travel more than two and a half miles along the curve just to get a foot above the x-axis.

FIGURE 1.42

The Sine Function

$$f(x) = \sin x$$
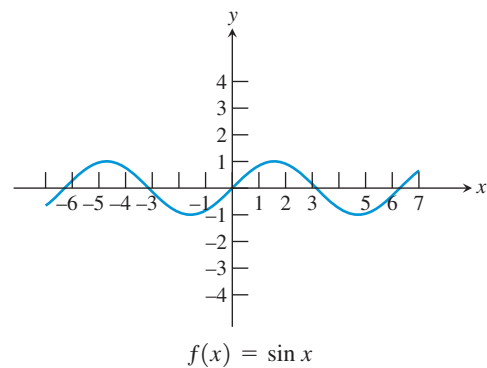
Interesting fact: This function and the sinus cavities in your head derive their names from a common root: the Latin word for "bay." This is due to a 12th-century mistake made by Robert of Chester, who translated a word incorrectly from an Arabic manuscript.

FIGURE 1.43

The Cosine Function

$$f(x) = \cos x$$

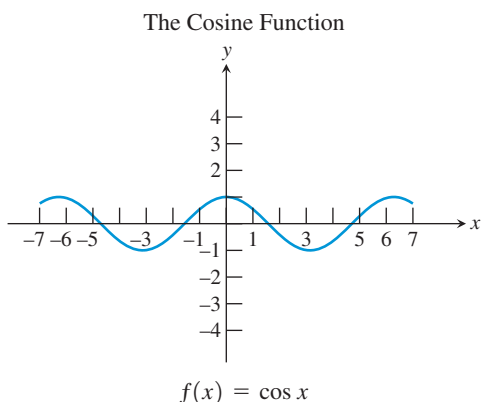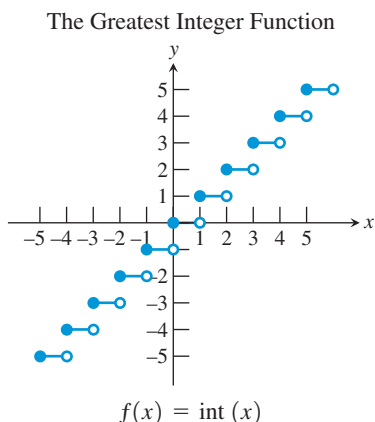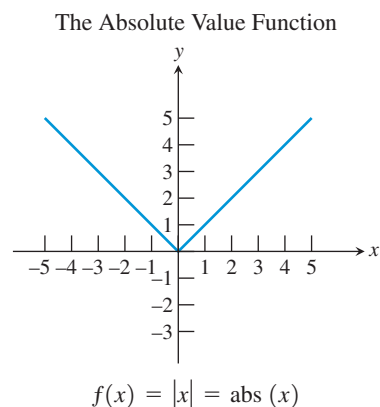Interesting fact: The local extrema of the cosine function occur exactly at the zeros of the sine function, and vice versa.

FIGURE 1.44

The Absolute Value Function

$$f(x) = |x| = \text{abs}(x)$$

Interesting fact: This function has an abrupt change of direction (a "corner") at the origin, while our other functions are all "smooth" on their domains.

FIGURE 1.45

The Greatest Integer Function
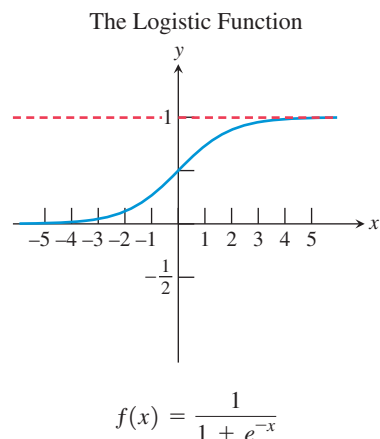

$$f(x) = \text{int}(x)$$

Interesting fact: This function has a jump discontinuity at every integer value of x. Similar-looking functions are called *step functions.*

FIGURE 1.46

The Logistic Function

$$f(x) = \frac{1}{1 + e^{-x}}$$

Interesting fact: There are two horizontal asymptotes, the x-axis and the line $y = 1$. This function provides a model for many applications in biology and business.

FIGURE 1.47

EXAMPLE 1   Looking for Domains

(a) Nine of the functions have domain the set of all real numbers. Which three do not?

(b) One of the functions has domain the set of all reals except 0. Which function is it, and why isn't zero in its domain?

(c) Which two functions have no negative numbers in their domains? Of these two, which one is defined at zero?

SOLUTION

(a) Imagine dragging a vertical line along the x-axis. If the function has domain the set of all real numbers, then the line will always intersect the graph. The intersection might occur off screen, but the TRACE function on the calculator will show the y-coordinate if there is one. Looking at the graphs in Figures 1.39, 1.40, and 1.42, we conjecture that there are vertical lines that do not intersect

(continued)

[–3.7, 5.7] by [–3.1, 3.1]

(a)

X=–2   Y=

[–3.7, 5.7] by [–3.1, 3.1]

(b)

X=0   Y=
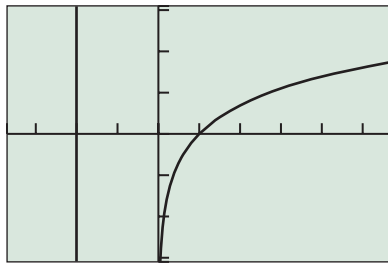
[–4.7, 4.7] by [–3.1, 3.1]

(c)

FIGURE 1.48 (a) A vertical line through −2 on the x-axis appears to miss the graph of $y = \ln x$. (b) A TRACE confirms that −2 is not in the domain. (c) A TRACE at $x = 0$ confirms that 0 is not in the domain of $y = 1/x$. (Example 1)
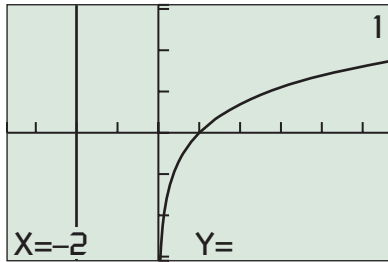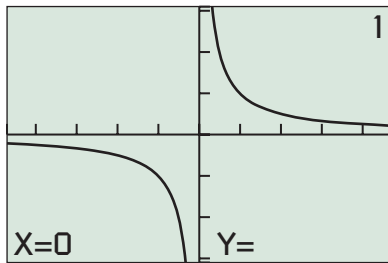
the curve. A TRACE at the suspected x-coordinates confirms our conjecture (Figure 1.48). The functions are $y = 1/x$, $y = \sqrt{x}$, and $y = \ln x$.

(b) The function $y = 1/x$, with a vertical asymptote at $x = 0$, is defined for all real numbers except 0. This is explained algebraically by the fact that division by zero is undefined.

(c) The functions $y = \sqrt{x}$ and $y = \ln x$ have no negative numbers in their domains. (We already knew that about the square root function.) While 0 is in the domain of $y = \sqrt{x}$, we can see by tracing that it is not in the domain of $y = \ln x$. We will see the algebraic reason for this in Chapter 3.

Now try Exercise 13.

EXAMPLE 2   Looking for Continuity

Only two of twelve functions have points of discontinuity. Are these points in the domain of the function?

SOLUTION All of the functions have continuous, unbroken graphs except for $y = 1/x$, and $y = \text{int } (x)$.

The graph of $y = 1/x$ clearly has an infinite discontinuity at $x = 0$ (Figure 1.39). We saw in Example 1 that 0 is not in the domain of the function. Since $y = 1/x$ is continuous for every point *in its domain*, it is called a **continuous function**.

The graph of $y = \text{int } (x)$ has a discontinuity at every integer value of x (Figure 1.46). Since this function has discontinuities at points in its domain, it is *not* a continuous function.

Now try Exercise 15.

EXAMPLE 3   Looking for Boundedness

Only three of the twelve basic functions are bounded (above and below). Which three?

SOLUTION A function that is bounded must have a graph that lies entirely between two horizontal lines. The sine, cosine, and logistic functions have this property (Figure 1.49). It looks like the graph of $y = \sqrt{x}$ might also have this property, but we know that the end behavior of the square root function is unbounded:

$\lim_{x \to \infty} \sqrt{x} = \infty$, so it is really only bounded below. You will learn in Chapter 4 why the sine and cosine functions are bounded.

Now try Exercise 17.

[–2π, 2π] by [–4, 4]

(a)
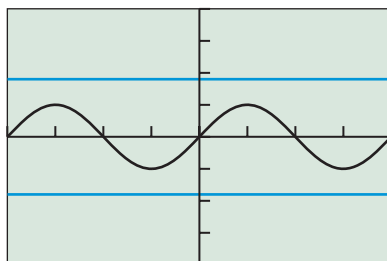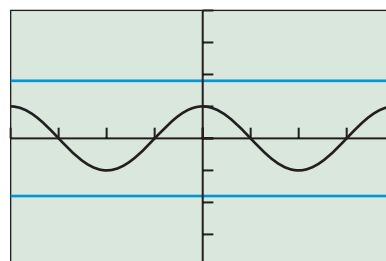
[–2π, 2π] by [–4, 4]

(b)
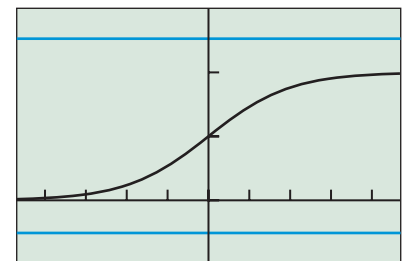
[–4.7, 4.7] by [–0.5, 1.5]

(c)

FIGURE 1.49 The graphs of $y = \sin x$, $y = \cos x$, and $y = 1/(1 + e^{-x})$ lie entirely between two horizontal lines and are therefore graphs of bounded functions. (Example 3)

EXAMPLE 4   Looking for Symmetry

Three of the twelve basic functions are even. Which are they?

SOLUTION   Recall that the graph of an even function is symmetric with respect to the y-axis. Three of the functions exhibit the required symmetry: $y = x^2$, $y = \cos x$, and $y = |x|$ (Figure 1.50). *Now try Exercise 19.*

[–4.7, 4.7] by [–1.1, 5.1]	[–2π, 2π] by [–4, 4]	[–4.7, 4.7] by [–1.1, 5.1]
(a)	(b)	(c)
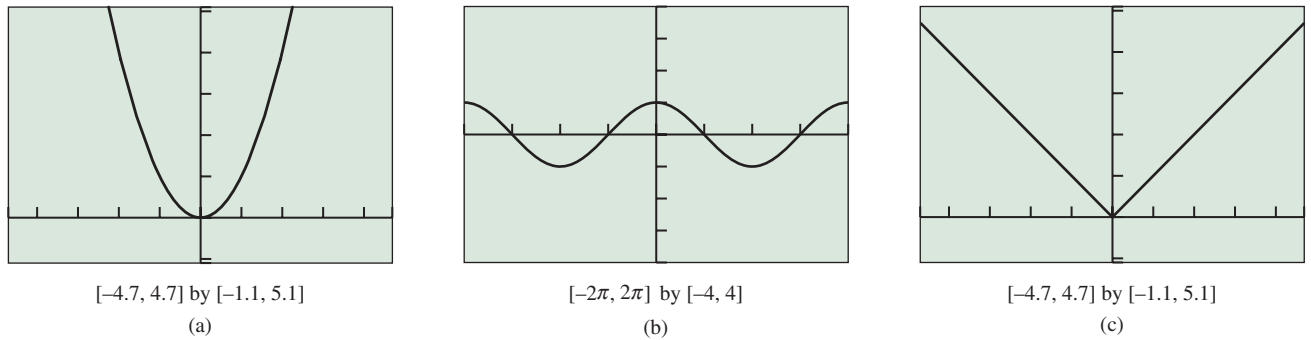
FIGURE 1.50 The graphs of $y = x^2$, $y = \cos x$, and $y = |x|$ are symmetric with respect to the y-axis, indicating that the functions are even. (Example 4)

Σ Analyzing Functions Graphically

We could continue to explore the twelve basic functions as in the first four examples, but we also want to make the point that there is no need to restrict ourselves to the basic twelve. We can alter the basic functions slightly and see what happens to their graphs, thereby gaining further visual insights into how functions behave.

EXAMPLE 5   Analyzing a Function Graphically

Graph the function $y = (x - 2)^2$. Then answer the following questions:

(a) On what interval is the function increasing? On what interval is it decreasing?

(b) Is the function odd, even, or neither?

(c) Does the function have any extrema?

(d) How does the graph relate to the graph of the basic function $y = x^2$?

SOLUTION   The graph is shown in Figure 1.51.

[–4.7, 4.7] by [–1.1, 5.1]

FIGURE 1.51 The graph of $y = (x - 2)^2$. (Example 5)

(continued)

(a) The function is increasing if its graph is headed upward as it moves from left to right. We see that it is increasing on the interval $[2, \infty)$. The function is decreasing if its graph is headed downward as it moves from left to right. We see that it is decreasing on the interval $(-\infty, 2]$.
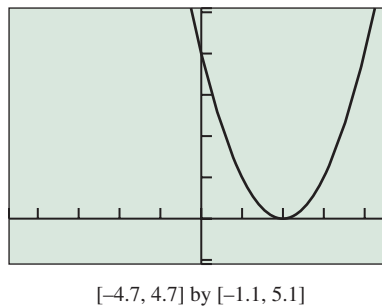
(b) The graph is not symmetric with respect to the y-axis, nor is it symmetric with respect to the origin. The function is neither.

(c) Yes, we see that the function has a minimum value of 0 at $x = 2$. (This is easily confirmed by the algebraic fact that $(x - 2)^2 \geq 0$ for all x.)

(d) We see that the graph of $y = (x - 2)^2$ is just the graph of $y = x^2$ moved two units to the right.   *Now try Exercise 35.*

EXPLORATION 1   Looking for Asymptotes

1. Two of the basic functions have vertical asymptotes at $x = 0$. Which two?

2. Form a new function by adding these functions together. Does the new function have a vertical asymptote at $x = 0$?

3. Three of the basic functions have horizontal asymptotes at $y = 0$. Which three?

4. Form a new function by adding these functions together. Does the new function have a horizontal asymptote at $y = 0$?

5. Graph $f(x) = 1/x$, $g(x) = 1/(2x^2 - x)$, and $h(x) = f(x) + g(x)$. Does $h(x)$ have a vertical asymptote at $x = 0$?

EXAMPLE 6   Identifying a Piecewise-Defined Function

Which of the twelve basic functions has the following **piecewise** definition over separate intervals of its domain?

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

SOLUTION   You may recognize this as the definition of the absolute value function (Chapter P). Or, you can reason that the graph of this function must look just like the line $y = x$ to the right of the y-axis, but just like the graph of the line $y = -x$ to the left of the y-axis. That is a perfect description of the absolute value graph in Figure 1.45. Either way, we recognize this as a piecewise definition of $f(x) = |x|$.   *Now try Exercise 45.*

EXAMPLE 7   Defining a Function Piecewise

Using basic functions from this section, construct a piecewise definition for the function whose graph is shown in Figure 1.52. Is your function continuous?

SOLUTION   This appears to be the graph of $y = x^2$ to the left of $x = 0$ and the graph of $y = \sqrt{x}$ to the right of $x = 0$. We can therefore define it piecewise as

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } x > 0. \end{cases}$$

The function is continuous.   *Now try Exercise 47.*

FIGURE 1.52   A piecewise-defined function. (Example 7)

You can go a long way toward understanding a function's behavior by looking at its graph. We will continue that theme in the exercises and then revisit it throughout the book. However, you can't go *all* the way toward understanding a function by looking at its graph, as Example 8 shows.
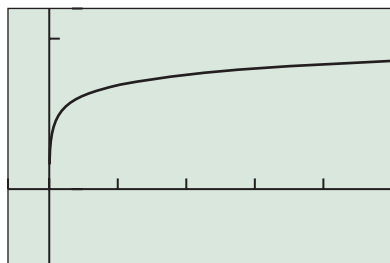
[–600, 5000] by [–5, 12]

FIGURE 1.53 The graph of $y = \ln x$ still appears to have a horizontal asymptote, despite the much larger window than in Figure 1.42. (Example 8)

EXAMPLE 8 Looking for a Horizontal Asymptote

Does the graph of $y = \ln x$ (see Figure 1.42) have a horizontal asymptote?

SOLUTION In Figure 1.42 it certainly *looks* like there is a horizontal asymptote that the graph is approaching from below. If we choose a much larger window (Figure 1.53), it still looks that way. In fact, we could zoom out on this function all day long and it would *always* look like it is approaching some horizontal asymptote— but it is not. We will show algebraically in Chapter 3 that the end behavior of this function is $\lim_{x \to \infty} \ln x = \infty$, so its graph must eventually rise above the level of any horizontal line. That rules out any horizontal asymptote, even though there is no *visual* evidence of that fact that we can see by looking at its graph.

Now try Exercise 55.

EXAMPLE 9 Analyzing a Function

Give a complete analysis of the basic function $f(x) = |x|$.

SOLUTION
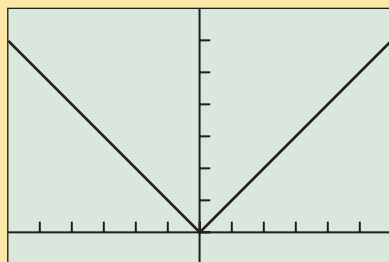
[–6, 6] by [–1, 7]

FIGURE 1.54 The graph of $f(x) = |x|$.

BASIC FUNCTION The Absolute Value Function

$f(x) = |x|$

Domain: All reals

Range: $[0, \infty)$

Continuous

Decreasing on $(-\infty, 0]$; increasing on $[0, \infty)$

Symmetric with respect to the y-axis (an even function)

Bounded below

Local minimum at $(0, 0)$

No horizontal asymptotes

No vertical asymptotes

End behavior: $\lim_{x \to -\infty} |x| = \infty$ and $\lim_{x \to \infty} |x| = \infty$

Now try Exercise 67.

QUICK REVIEW 1.3 *(For help, go to Sections P.1, P.2, 3.1, and 3.3.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–10, evaluate the expression without using a calculator.

1. $|-59.34|$ **2.** $|5 - \pi|$

3. $|\pi - 7|$ **4.** $\sqrt{(-3)^2}$

5. $\ln(1)$ **6.** e^0

**7.** $(\sqrt[3]{3})^3$ **8.** $\sqrt[3]{(-15)^3}$

9. $\sqrt[3]{-8^2}$ **10.** $|1 - \pi| - \pi$

SECTION 1.3 EXERCISES

In Exercises 1–12, each graph is a slight variation on the graph of one of the twelve basic functions described in this section. Match the graph to one of the twelve functions (a)–(l) and then support your answer by checking the graph on your calculator. (All graphs are shown in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$.)

(a) $y = -\sin x$ (b) $y = \cos x + 1$ (c) $y = e^x - 2$

(d) $y = (x + 2)^3$ (e) $y = x^3 + 1$ (f) $y = (x - 1)^2$

(g) $y = |x| - 2$ (h) $y = -1/x$ (i) $y = -x$

(j) $y = -\sqrt{x}$ (k) $y = \text{int}(x + 1)$

(l) $y = 2 - 4/(1 + e^{-x})$

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

12.
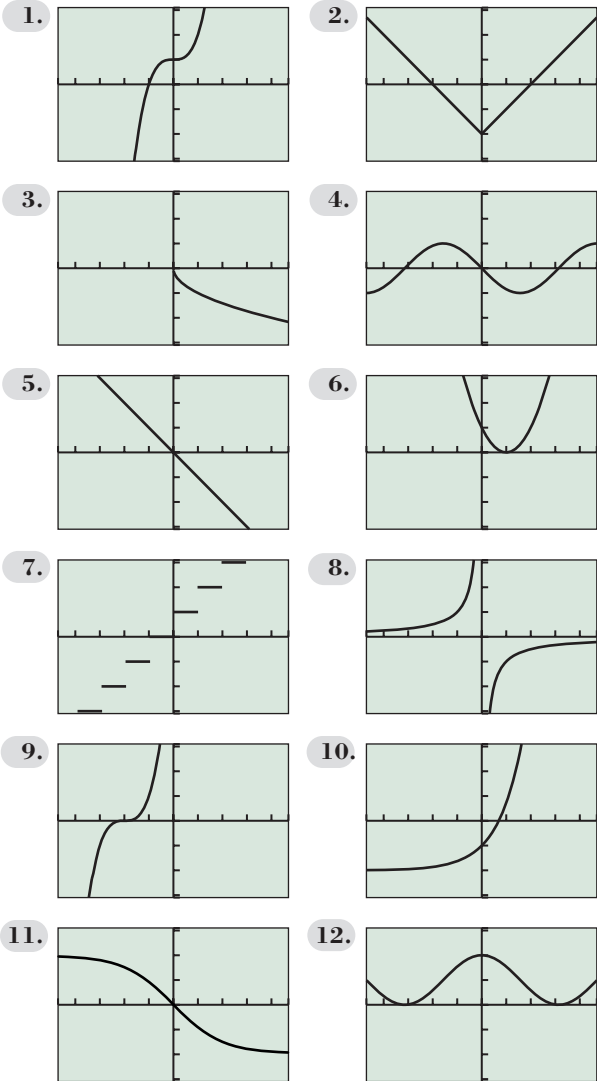
In Exercises 13–18, identify which of Exercises 1–12 display functions that fit the description given.

13. The function whose domain excludes zero

14. The function whose domain consists of all nonnegative real numbers

15. The two functions that have at least one point of discontinuity

16. The function that is not a *continuous function*

17. The six functions that are bounded below

18. The four functions that are bounded above

In Exercises 19–28, identify which of the twelve basic functions fit the description given.

19. The four functions that are odd

20. The six functions that are increasing on their entire domains

21. The three functions that are decreasing on the interval $(-\infty, 0)$

22. The three functions with infinitely many local extrema

23. The three functions with no zeros

24. The three functions with range {all real numbers}

25. The four functions that do *not* have end behavior
$$\lim_{x \to +\infty} f(x) = +\infty$$

26. The three functions with end behavior $\lim_{x \to -\infty} f(x) = -\infty$

27. The four functions whose graphs look the same when turned upside-down and flipped about the *y*-axis

28. The two functions whose graphs are identical except for a horizontal shift

In Exercises 29–34, use your graphing calculator to produce a graph of the function. Then determine the domain and range of the function by looking at its graph.

29. $f(x) = x^2 - 5$ 30. $g(x) = |x - 4|$

31. $h(x) = \ln (x + 6)$ 32. $k(x) = 1/x + 3$

33. $s(x) = \text{int} (x/2)$ 34. $p(x) = (x + 3)^2$

In Exercises 35–42, graph the function. Then answer the following questions:

(a) On what interval, if any, is the function increasing? Decreasing?

(b) Is the function odd, even, or neither?

(c) Give the function's extrema, if any.

(d) How does the graph relate to a graph of one of the twelve basic functions?

35. $r(x) = \sqrt{x - 10}$ 36. $f(x) = \sin (x) + 5$

37. $f(x) = 3/(1 + e^{-x})$ 38. $q(x) = e^x + 2$

39. $h(x) = |x| - 10$ 40. $g(x) = 4 \cos (x)$

41. $s(x) = |x - 2|$ 42. $f(x) = 5 - \text{abs} (x)$

43. Find the horizontal asymptotes for the graph shown in Exercise 11.

44. Find the horizontal asymptotes for the graph of $f(x)$ in Exercise 37.

In Exercises 45–52, sketch the graph of the piecewise-defined function. (Try doing it without a calculator.) In each case, give any points of discontinuity.

45. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$

46. $g(x) = \begin{cases} x^3 & \text{if } x \le 0 \\ e^x & \text{if } x > 0 \end{cases}$

47. $h(x) = \begin{cases} |x| & \text{if } x < 0 \\ \sin x & \text{if } x \ge 0 \end{cases}$

48. $w(x) = \begin{cases} 1/x & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \ge 0 \end{cases}$

49. $f(x) = \begin{cases} \cos x & \text{if } x \le 0 \\ e^x & \text{if } x > 0 \end{cases}$

50. $g(x) = \begin{cases} |x| & \text{if } x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$

51. $f(x) = \begin{cases} -3 - x & \text{if } x \le 0 \\ 1 & \text{if } 0 < x < 1 \\ x^2 & \text{if } x \ge 1 \end{cases}$

52. $f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ |x| & \text{if } -1 \le x < 1 \\ \text{int}(x) & \text{if } x \ge 1 \end{cases}$

53. Writing to Learn   The function $f(x) = \sqrt{x^2}$ is one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it is.

(b) Explain algebraically why the two functions are equal.

54. Uncovering Hidden Behavior   The function $g(x) = \sqrt{x^2 + 0.0001} - 0.01$ is *not* one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it appears to be.

(b) Verify numerically that it is not the basic function that it appears to be.

55. Writing to Learn   The function $f(x) = \ln(e^x)$ is one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it is.

(b) Explain how the equivalence of the two functions in (a) shows that the natural logarithm function is *not* bounded above (even though it *appears* to be bounded above in Figure 1.42).

56. Writing to Learn   Let $f(x)$ be the function that gives the cost, in cents, to mail a first-class package that weighs x ounces. In August of 2009, the cost was \$1.22 for a package that weighed up to 1 ounce, plus 17 cents for each additional ounce or portion thereof (up to 13 ounces). (*Source: United States Postal Service.*)

(a) Sketch a graph of $f(x)$.

(b) How is this function similar to the greatest integer function? How is it different?

Packages

Weight Not Over	Price
1 ounce	\$1.22
2 ounces	\$1.39
3 ounces	\$1.56
4 ounces	\$1.73
5 ounces	\$1.90
6 ounces	\$2.07
7 ounces	\$2.24
8 ounces	\$2.41
9 ounces	\$2.58
10 ounces	\$2.75
11 ounces	\$2.92
12 ounces	\$3.09
13 ounces	\$3.26

57. Analyzing a Function   Set your calculator to DOT mode and graph the greatest integer function, $y = \text{int}(x)$, in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$. Then complete the following analysis.

BASIC FUNCTION
The Greatest Integer Function

$f(x) = \text{int } x$

Domain:
Range:
Continuity:
Increasing/decreasing behavior:
Symmetry:
Boundedness:
Local extrema:
Horizontal asymptotes:
Vertical asymptotes:
End behavior:

Standardized Test Questions

58. True or False   The greatest integer function has an inverse function. Justify your answer.

59. True or False   The logistic function has two horizontal asymptotes. Justify your answer.

In Exercises 60–63, you may use a graphing calculator to answer the question.

60. Multiple Choice   Which function has range {all real numbers}?

(A) $f(x) = 4 + \ln x$

(B) $f(x) = 3 - 1/x$

(C) $f(x) = 5/(1 + e^{-x})$

(D) $f(x) = \text{int}(x - 2)$

(E) $f(x) = 4 \cos x$

61. Multiple Choice   Which function is bounded both above and below?

(A) $f(x) = x^2 - 4$

(B) $f(x) = (x - 3)^3$

(C) $f(x) = 3e^x$

(D) $f(x) = 3 + 1/(1 + e^{-x})$

(E) $f(x) = 4 - |x|$

62. Multiple Choice   Which of the following is the same as the restricted-domain function $f(x) = \text{int}(x), 0 \leq x < 2$?

(A) $f(x) = \begin{cases} 0 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x = 1 \\ 2 & \text{if } 1 < x < 2 \end{cases}$

(B) $f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } 0 < x \leq 1 \\ 2 & \text{if } 1 < x < 2 \end{cases}$

(C) $f(x) = \begin{cases} 0 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 \leq x < 2 \end{cases}$

(D) $f(x) = \begin{cases} 1 & \text{if } 0 \leq x < 1 \\ 2 & \text{if } 1 \leq x < 2 \end{cases}$

(E) $f(x) = \begin{cases} x & \text{if } 0 \leq x < 1 \\ 1 + x & \text{if } 1 \leq x < 2 \end{cases}$

63. Multiple Choice Increasing Functions   Which function is increasing on the interval $(-\infty, \infty)$?

(A) $f(x) = \sqrt{3 + x}$

(B) $f(x) = \text{int}(x)$

(C) $f(x) = 2x^2$

(D) $f(x) = \sin x$

(E) $f(x) = 3/(1 + e^{-x})$

Explorations

64. Which Is Bigger?   For positive values of x, we wish to compare the values of the basic functions x^2, x, and $\sqrt{x}$.

(a) How would you order them from least to greatest?

(b) Graph the three functions in the viewing window $[0, 30]$ by $[0, 20]$. Does the graph confirm your response in (a)?

(c) Now graph the three functions in the viewing window $[0, 2]$ by $[0, 1.5]$.

(d) Write a careful response to the question in (a) that accounts for all positive values of x.

65. Odds and Evens   There are four odd functions and three even functions in the gallery of twelve basic functions. After multiplying these functions together pairwise in different combinations and exploring the graphs of the products, make a conjecture about the symmetry of:

(a) a product of two odd functions;

(b) a product of two even functions;

(c) a product of an odd function and an even function.

66. Group Activity   Assign to each student in the class the name of one of the twelve basic functions, but secretly so that no student knows the "name" of another. (The same function name could be given to several students, but all the functions should be used at least once.) Let each student make a one-sentence self-introduction to the class that reveals something personal "about who I am that really identifies me." The rest of the students then write down their guess as to the function's identity. Hints should be subtle and cleverly anthropomorphic. (For example, the absolute value function saying "I have a very sharp smile" is subtle and clever, while "I am absolutely valuable" is not very subtle at all.)

67. Pepperoni Pizzas   For a statistics project, a student counted the number of pepperoni slices on pizzas of various sizes at a local pizzeria, compiling the following table:

| Table 1.10		
Type of Pizza	Radius	Pepperoni Count
Personal	4″	12
Medium	6″	27
Large	7″	37
Extra large	8″	48

(a) Explain why the pepperoni count (P) ought to be proportional to the square of the radius (r).

(b) Assuming that $P = k \cdot r^2$, use the data pair $(4, 12)$ to find the value of k.

(c) Does the algebraic model fit the rest of the data well?

(d) Some pizza places have charts showing their kitchen staff how much of each topping should be put on each size of pizza. Do you think this pizzeria uses such a chart? Explain.

Extending the Ideas

68. Inverse Functions   Two functions are said to be *inverses* of each other if the graph of one can be obtained from the graph of the other by reflecting it across the line $y = x$. For example, the functions with the graphs shown below are inverses of each other:
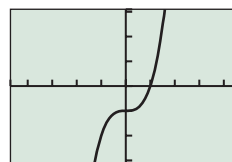
[–4.7, 4.7] by [–3.1, 3.1]
(a)
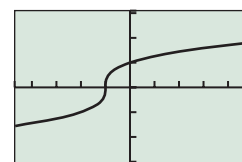
[–4.7, 4.7] by [–3.1, 3.1]
(b)

(a) Two of the twelve basic functions in this section are inverses of each other. Which are they?

(b) Two of the twelve basic functions in this section are their own inverses. Which are they?

(c) If you restrict the domain of one of the twelve basic functions to $[0, \infty)$, it becomes the inverse of another one. Which are they?

69. Identifying a Function by Its Properties

(a) Seven of the twelve basic functions have the property that $f(0) = 0$. Which five do not?

(b) Only one of the twelve basic functions has the property that $f(x + y) = f(x) + f(y)$ for all x and y in its domain. Which one is it?

(c) One of the twelve basic functions has the property that $f(x + y) = f(x)f(y)$ for all x and y in its domain. Which one is it?

(d) One of the twelve basic functions has the property that $f(xy) = f(x) + f(y)$ for all x and y in its domain. Which one is it?

(e) Four of the twelve basic functions have the property that $f(x) + f(-x) = 0$ for all x in their domains. Which four are they?