1.4 Building Functions from Functions

What you'll learn about

- Combining Functions Algebraically
- Composition of Functions
- Relations and Implicitly Defined Functions

... and why

Most of the functions that you will encounter in calculus and in real life can be created by combining or modifying other functions.

Combining Functions Algebraically

Knowing how a function is "put together" is an important first step when applying the tools of calculus. Functions have their own algebra based on the same operations we apply to real numbers (addition, subtraction, multiplication, and division). One way to build new functions is to apply these operations, using the following definitions.

DEFINITION Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions with intersecting domains. Then for all values of x in the intersection, the algebraic combinations of f and g are defined by the following rules:

Sum: $(f + g)(x) = f(x) + g(x)$

Difference: $(f - g)(x) = f(x) - g(x)$

Product: $(fg)(x) = f(x)g(x)$

Quotient: $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$, provided $g(x) \neq 0$

In each case, the domain of the new function consists of all numbers that belong to both the domain of f and the domain of g. As noted, the zeros of the denominator are excluded from the domain of the quotient.

Euler's function notation works so well in the above definitions that it almost obscures what is really going on. The "+" in the expression "$(f + g)(x)$" stands for a brand new operation called *function addition*. It builds a new function, $f + g$, from the given functions f and g. Like any function, $f + g$ is defined by what it does: It takes a domain value x and returns a range value $f(x) + g(x)$. Note that the "+" sign in "$f(x) + g(x)$" *does* stand for the familiar operation of real number addition. So, with the same symbol taking on different roles on either side of the equal sign, there is more to the above definitions than first meets the eye.

Fortunately, the definitions are easy to apply.

EXAMPLE 1 Defining New Functions Algebraically

Let $f(x) = x^2$ and $g(x) = \sqrt{x + 1}$.

Find formulas for the functions $f + g$, $f - g$, fg, f/g, and gg. Give the domain of each.

(continued)

SOLUTION We first determine that f has domain all real numbers and that g has domain $[-1, \infty)$. These domains overlap, the intersection being the interval $[-1, \infty)$. So:

$$(f+g)(x) = f(x) + g(x) = x^2 + \sqrt{x+1} \quad \text{with domain } [-1, \infty).$$
$$(f-g)(x) = f(x) - g(x) = x^2 - \sqrt{x+1} \quad \text{with domain } [-1, \infty).$$
$$(fg)(x) = f(x)g(x) = x^2\sqrt{x+1} \quad \text{with domain } [-1, \infty).$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{\sqrt{x+1}} \quad \text{with domain } (-1, \infty).$$
$$(gg)(x) = g(x)g(x) = (\sqrt{x+1})^2 \quad \text{with domain } [-1, \infty).$$

Note that we could express $(gg)(x)$ more simply as $x + 1$. That would be fine, but the simplification would not change the fact that the domain of gg is (by definition) the interval $[-1, \infty)$. Under other circumstances the function $h(x) = x + 1$ would have domain all real numbers, but under these circumstances it cannot; it is a product of two functions with restricted domains. ***Now try Exercise 3.***

Composition of Functions

It is not hard to see that the function $\sin(x^2)$ is built from the basic functions $\sin x$ and x^2, but the functions are not put together by addition, subtraction, multiplication, or division. Instead, the two functions are combined by simply applying them in order—first the squaring function, then the sine function. This operation for combining functions, which has no counterpart in the algebra of real numbers, is called *function composition*.

DEFINITION Composition of Functions

Let f and g be two functions such that the domain of f intersects the range of g. The **composition *f* of g**, denoted $f \circ g$, is defined by the rule

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ consists of all x-values in the domain of g that map to $g(x)$-values in the domain of f. (See Figure 1.55.)

The composition g of f, denoted $g \circ f$, is defined similarly. In most cases $g \circ f$ and $f \circ g$ are different functions. (In the language of algebra, "function composition is not commutative.")

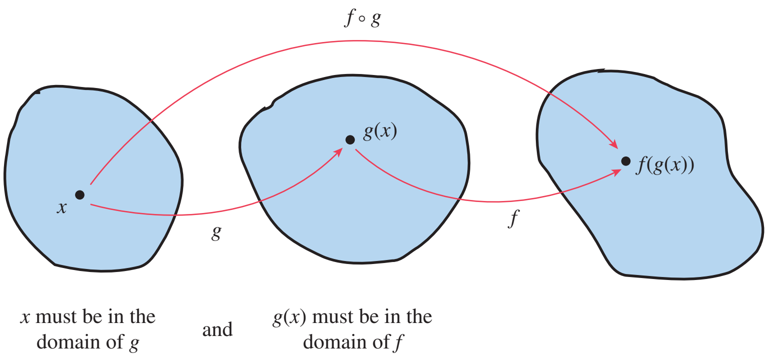


FIGURE 1.55 In the composition $f \circ g$, the function g is applied first and then f. This is the reverse of the order in which we read the symbols.

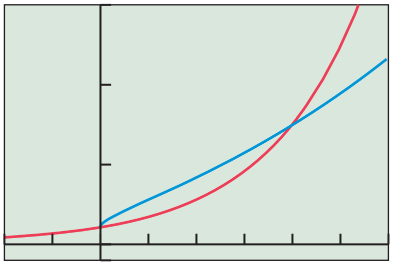
EXAMPLE 2 Composing Functions

Let $f(x) = e^x$ and $g(x) = \sqrt{x}$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ and verify numerically that the functions $f \circ g$ and $g \circ f$ are not the same.

SOLUTION

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = e^{\sqrt{x}}$$
$$(g \circ f)(x) = g(f(x)) = g(e^x) = \sqrt{e^x}$$

One verification that these functions are not the same is that they have different domains: $f \circ g$ is defined only for $x \geq 0$, while $g \circ f$ is defined for all real numbers. We could also consider their graphs (Figure 1.56), which agree only at $x = 0$ and $x = 4$.



[–2, 6] by [–1, 15]

FIGURE 1.56 The graphs of $y = e^{\sqrt{x}}$ and $y = \sqrt{e^x}$ are not the same. (Example 2)

Finally, the graphs suggest a numerical verification: Find a single value of x for which $f(g(x))$ and $g(f(x))$ give different values. For example, $f(g(1)) = e$ and $g(f(1)) = \sqrt{e}$. The graph helps us to make a judicious choice of x. You do not want to check the functions at $x = 0$ and $x = 4$ and conclude that they are the same!

Now try Exercise 15.

EXPLORATION 1 Composition Calisthenics

One of the f functions in column B can be composed with one of the g functions in column C to yield each of the basic $f \circ g$ functions in column A. Can you match the columns successfully without a graphing calculator? If you are having trouble, try it with a graphing calculator.

A	B	C
$f \circ g$	f	g
x	$x - 3$	$x^{0.6}$
x^2	$2x - 3$	x^2
$\lvert x\rvert$	$\sqrt{x}$	$\dfrac{(x-2)(x+2)}{2}$
x^3	x^5	$\ln(e^3 x)$
$\ln x$	$\lvert 2x + 4\rvert$	$\dfrac{x}{2}$
$\sin x$	$1 - 2x^2$	$\dfrac{x+3}{2}$
$\cos x$	$2 \sin x \cos x$	$\sin\left(\dfrac{x}{2}\right)$

EXAMPLE 3 Finding the Domain of a Composition

Let $f(x) = x^2 - 1$ and let $g(x) = \sqrt{x}$. Find the domains of the composite functions

(a) $g \circ f$ **(b)** $f \circ g$.

SOLUTION

(a) We compose the functions in the order specified:

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= \sqrt{x^2 - 1}\end{aligned}$$

For x to be in the domain of $g \circ f$, we must first find $f(x) = x^2 - 1$, which we can do for all real x. Then we must take the square root of the result, which we can only do for nonnegative values of $x^2 - 1$.

Therefore, the domain of $g \circ f$ consists of all real numbers for which $x^2 - 1 \geq 0$, namely the union $(-\infty, -1] \cup [1, \infty)$.

(b) Again, we compose the functions in the order specified:

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= (\sqrt{x})^2 - 1\end{aligned}$$

For x to be in the domain of $f \circ g$, we must first be able to find $g(x) = \sqrt{x}$, which we can only do for nonnegative values of x. Then we must be able to square the result and subtract 1, which we can do for all real numbers. Therefore, the domain of $f \circ g$ consists of the interval $[0, \infty)$.

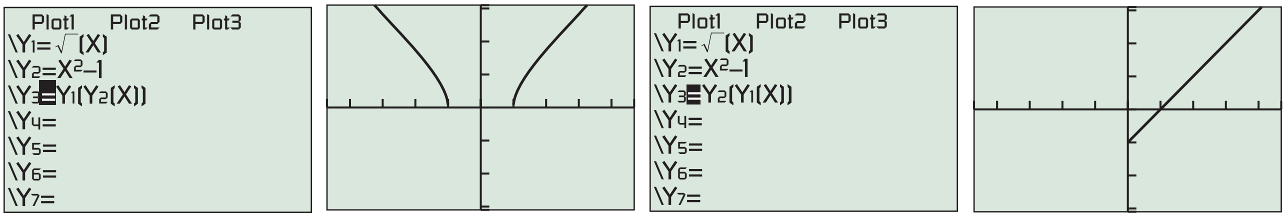
Support Graphically

We can graph the composition functions to see if the grapher respects the domain restrictions. The screen to the left of each graph shows the setup in the "Y =" editor. Figure 1.57b shows the graph of $y = (g \circ f)(x)$, while Figure 1.57d shows the graph of $y = (f \circ g)(x)$. The graphs support our algebraic work quite nicely.

Now try Exercise 17.

Caution

We might choose to express $(f \circ g)$ more simply as $x - 1$. However, you must remember that the composition is restricted to the domain of $g(x) = \sqrt{x}$, or $[0, \infty]$. The domain of $x - 1$ is all real numbers. It is a good idea to work out the domain of a composition before you simplify the expression for $f(g(x))$. One way to simplify and maintain the restriction on the domain in Example 3 is to write $(f \circ g)(x) = x - 1, x \geq 0$.



(a) (b) [–4.7, 4.7] by [–3.1, 3.1] (c) (d) [–4.7, 4.7] by [–3.1, 3.1]

FIGURE 1.57 The functions Y1 and Y2 are composed to get the graphs of $y = (g \circ f)(x)$ and $y = (f \circ g)(x)$, respectively. The graphs support our conclusions about the domains of the two composite functions. (Example 3)

In Examples 2 and 3 two functions were *composed* to form new functions. There are times in calculus when we need to reverse the process. That is, we may begin with a function h and *decompose* it by finding functions whose composition is h.

EXAMPLE 4 Decomposing Functions

For each function h, find functions f and g such that $h(x) = f(g(x))$.

(a) $h(x) = (x + 1)^2 - 3(x + 1) + 4$

(b) $h(x) = \sqrt{x^3 + 1}$

(continued)

SOLUTION

(a) We can see that h is quadratic in $x + 1$. Let $f(x) = x^2 - 3x + 4$ and let $g(x) = x + 1$. Then

$$h(x) = f(g(x)) = f(x+1) = (x+1)^2 - 3(x+1) + 4.$$

(b) We can see that h is the square root of the function $x^3 + 1$. Let $f(x) = \sqrt{x}$ and let $g(x) = x^3 + 1$. Then

$$h(x) = f(g(x)) = f(x^3+1) = \sqrt{x^3+1}.$$

Now try Exercise 25.

There is often more than one way to decompose a function. For example, an alternate way to decompose $h(x) = \sqrt{x^3+1}$ in Example 4b is to let $f(x) = \sqrt{x+1}$ and let $g(x) = x^3$. Then $h(x) = f(g(x)) = f(x^3) = \sqrt{x^3+1}$.

EXAMPLE 5 Modeling with Function Composition

In the medical procedure known as angioplasty, doctors insert a catheter into a heart vein (through a large peripheral vein) and inflate a small, spherical balloon on the tip of the catheter. Suppose the balloon is inflated at a constant rate of 44 cubic millimeters per second (Figure 1.58).

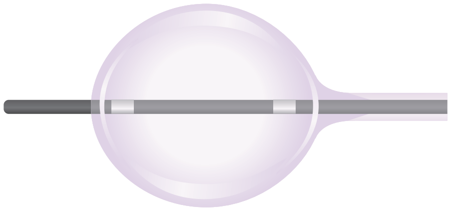


FIGURE 1.58 (Example 5)

(a) Find the volume after t seconds.

(b) When the volume is V, what is the radius r?

(c) Write an equation that gives the radius r as a function of the time. What is the radius after 5 seconds?

SOLUTION

(a) After t seconds, the volume will be $44t$.

(b) Solve Algebraically

$$\frac{4}{3}\pi r^3 = v$$

$$r^3 = \frac{3v}{4\pi}$$

$$r = \sqrt[3]{\frac{3v}{4\pi}}$$

(c) Substituting $44t$ for V gives $r = \sqrt[3]{\frac{3 \cdot 44t}{4\pi}}$ or $r = \sqrt[3]{\frac{33t}{\pi}}$. After 5 seconds, the radius will be $r = \sqrt[3]{\frac{33 \cdot 5}{\pi}} \approx 3.74$ mm.

Now try Exercise 31.

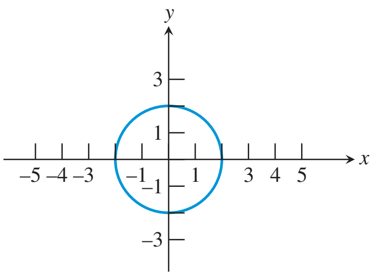


FIGURE 1.59 A circle of radius 2 centered at the origin. This set of ordered pairs (x, y) defines a *relation* that is not a function, because the graph fails the vertical line test.

Relations and Implicitly Defined Functions

There are many useful curves in mathematics that fail the vertical line test and therefore are not graphs of functions. One such curve is the circle in Figure 1.59. While y is not related to x as a function in this instance, there is certainly some sort of relationship going on. In fact, not only does the shape of the graph show a significant *geometric* relationship among the points, but the ordered pairs (x, y) exhibit a significant *algebraic* relationship as well: They consist exactly of the solutions to the equation $x^2 + y^2 = 4$.

Graphing Relations

Relations that are not functions are often not easy to graph. We will study some special cases later in the course (circles, ellipses, etc.), but some simple-looking relations like those in Example 6 are difficult to graph. Nor do our calculators help much, because the equation cannot be put into "Y1=" form. Interestingly, we *do* know that the graph of the relation in Example 6, whatever it looks like, fails the vertical line test.

The general term for a set of ordered pairs (x, y) is a **relation**. If the relation happens to relate a *single* value of y to each value of x, then the relation is also a function and its graph will pass the vertical line test. In the case of the circle with equation $x^2 + y^2 = 4$, both $(0, 2)$ and $(0, -2)$ are in the relation, so y is not a function of x.

EXAMPLE 6 Verifying Pairs in a Relation

Determine which of the ordered pairs $(2, -5)$, $(1, 3)$, and $(2, 1)$ are in the relation defined by $x^2y + y^2 = 5$. Is the relation a function?

SOLUTION We simply substitute the x- and y-coordinates of the ordered pairs into $x^2y + y^2$ and see if we get 5.

$(2, -5)$:	$(2)^2(-5) + (-5)^2 = 5$	Substitute $x = 2$, $y = -5$.
$(1, 3)$:	$(1)^2(3) + (3)^2 = 12 \neq 5$	Substitute $x = 1$, $y = 3$.
$(2, 1)$:	$(2)^2(1) + (1)^2 = 5$	Substitute $x = 2$, $y = 1$.

So, $(2, -5)$ and $(2, 1)$ are in the relation, but $(1, 3)$ is not.

Since the equation relates two different y-values (-5 and 1) to the same x-value (2), the relation cannot be a function. ***Now try Exercise 35.***

Let us revisit the circle $x^2 + y^2 = 4$. While it is not a function itself, we can split it into two equations that *do* define functions, as follows:

$$x^2 + y^2 = 4$$
$$y^2 = 4 - x^2$$
$$y = +\sqrt{4 - x^2} \text{ or } y = -\sqrt{4 - x^2}$$

The graphs of these two functions are, respectively, the upper and lower semicircles of the circle in Figure 1.59. They are shown in Figure 1.60. Since all the ordered pairs in either of these functions satisfy the equation $x^2 + y^2 = 4$, we say that the relation given by the equation defines the two functions **implicitly**.

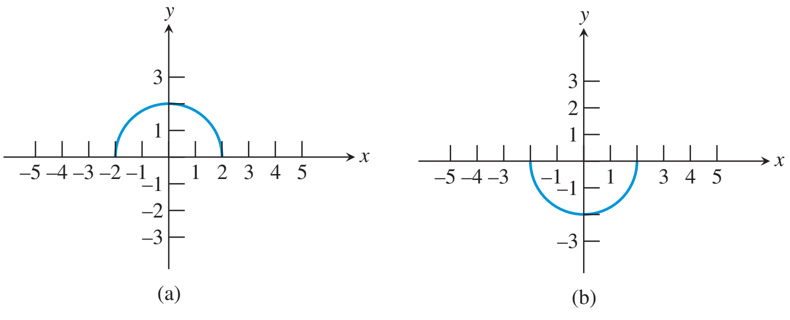


FIGURE 1.60 The graphs of (a) $y = +\sqrt{4 - x^2}$ and (b) $y = -\sqrt{4 - x^2}$. In each case, y is defined as a function of x. These two functions are defined *implicitly* by the relation $x^2 + y^2 = 4$.

EXAMPLE 7 Using Implicitly Defined Functions

Describe the graph of the relation $x^2 + 2xy + y^2 = 1$.

SOLUTION This looks like a difficult task at first, but notice that the expression on the left of the equal sign is a factorable trinomial. This enables us to split the relation into two implicitly defined functions as follows:

$$\begin{aligned} x^2 + 2xy + y^2 &= 1 \\ (x + y)^2 &= 1 && \text{Factor.} \\ x + y &= \pm 1 && \text{Extract square roots.} \\ x + y &= 1 \text{ or } x + y = -1 \\ y &= -x + 1 \text{ or } y = -x - 1 && \text{Solve for } y. \end{aligned}$$

The graph consists of two parallel lines (Figure 1.61), each the graph of one of the implicitly defined functions. *Now try Exercise 37.*

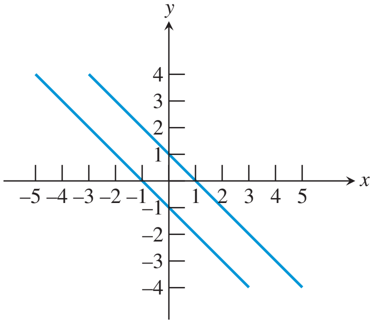


FIGURE 1.61 The graph of the relation $x^2 + 2xy + y^2 = 1$. (Example 7)

QUICK REVIEW 1.4 *(For help, go to Sections P.1, 1.2, and 1.3.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–10, find the domain of the function and express it in interval notation.

1. $f(x) = \dfrac{x - 2}{x + 3}$

2. $g(x) = \ln(x - 1)$

3. $f(t) = \sqrt{5 - t}$

4. $g(x) = \dfrac{3}{\sqrt{2x - 1}}$

5. $f(x) = \sqrt{\ln(x)}$

6. $h(x) = \sqrt{1 - x^2}$

7. $f(t) = \dfrac{t + 5}{t^2 + 1}$

8. $g(t) = \ln(|t|)$

9. $f(x) = \dfrac{1}{\sqrt{1 - x^2}}$

10. $g(x) = 2$

SECTION 1.4 EXERCISES

In Exercises 1–4, find formulas for the functions $f + g$, $f - g$, and fg. Give the domain of each.

1. $f(x) = 2x - 1;\ g(x) = x^2$

2. $f(x) = (x - 1)^2;\ g(x) = 3 - x$

3. $f(x) = \sqrt{x};\ g(x) = \sin x$

4. $f(x) = \sqrt{x + 5};\ g(x) = |x + 3|$

In Exercises 5–8, find formulas for f/g and g/f. Give the domain of each.

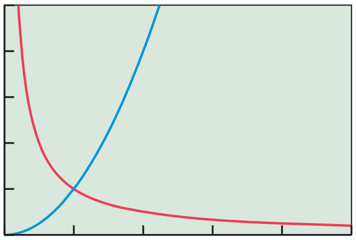
5. $f(x) = \sqrt{x + 3};\ g(x) = x^2$

6. $f(x) = \sqrt{x - 2};\ g(x) = \sqrt{x + 4}$

7. $f(x) = x^2;\ g(x) = \sqrt{1 - x^2}$

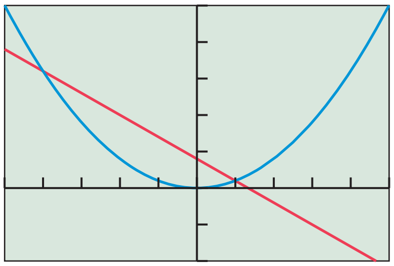
8. $f(x) = x^3;\ g(x) = \sqrt[3]{1 - x^3}$

9. $f(x) = x^2$ and $g(x) = 1/x$ are shown below in the viewing window $[0, 5]$ by $[0, 5]$. Sketch the graph of the sum $(f + g)(x)$ by adding the y-coordinates directly from the graphs. Then graph the sum on your calculator and see how close you came.



[0, 5] by [0, 5]

10. The graphs of $f(x) = x^2$ and $g(x) = 4 - 3x$ are shown in the viewing window $[-5, 5]$ by $[-10, 25]$. Sketch the graph of the difference $(f - g)(x)$ by subtracting the y-coordinates directly from the graphs. Then graph the difference on your calculator and see how close you came.



[–5, 5] by [–10, 25]

In Exercises 11–14, find $(f \circ g)(3)$ and $(g \circ f)(-2)$.

11. $f(x) = 2x - 3$; $g(x) = x + 1$

12. $f(x) = x^2 - 1$; $g(x) = 2x - 3$

13. $f(x) = x^2 + 4$; $g(x) = \sqrt{x + 1}$

14. $f(x) = \dfrac{x}{x + 1}$; $g(x) = 9 - x^2$

In Exercises 15–22, find $f(g(x))$ and $g(f(x))$. State the domain of each.

15. $f(x) = 3x + 2$; $g(x) = x - 1$

16. $f(x) = x^2 - 1$; $g(x) = \dfrac{1}{x - 1}$

17. $f(x) = x^2 - 2$; $g(x) = \sqrt{x + 1}$

18. $f(x) = \dfrac{1}{x - 1}$; $g(x) = \sqrt{x}$

19. $f(x) = x^2$; $g(x) = \sqrt{1 - x^2}$

20. $f(x) = x^3$; $g(x) = \sqrt[3]{1 - x^3}$

21. $f(x) = \dfrac{1}{2x}$; $g(x) = \dfrac{1}{3x}$

22. $f(x) = \dfrac{1}{x + 1}$; $g(x) = \dfrac{1}{x - 1}$

In Exercises 23–30, find $f(x)$ and $g(x)$ so that the function can be described as $y = f(g(x))$. (There may be more than one possible decomposition.)

23. $y = \sqrt{x^2 - 5x}$

24. $y = (x^3 + 1)^2$

25. $y = |3x - 2|$

26. $y = \dfrac{1}{x^3 - 5x + 3}$

27. $y = (x - 3)^5 + 2$

28. $y = e^{\sin x}$

29. $y = \cos(\sqrt{x})$

30. $y = (\tan x)^2 + 1$

31. Weather Balloons A high-altitude spherical weather balloon expands as it rises due to the drop in atmospheric pressure. Suppose that the radius r increases at the rate of 0.03 inch per second and that $r = 48$ inches at time $t = 0$. Determine an equation that models the volume V of the balloon at time t and find the volume when $t = 300$ seconds.

32. A Snowball's Chance Jake stores a small cache of 4-inch-diameter snowballs in the basement freezer, unaware that the freezer's self-defrosting feature will cause each snowball to lose about 1 cubic inch of volume every 40 days. He remembers them a year later (call it 360 days) and goes to retrieve them. What is their diameter then?

33. Satellite Photography A satellite camera takes a rectangle-shaped picture. The smallest region that can be photographed is a 5-km by 7-km rectangle. As the camera zooms out, the length l and width w of the rectangle increase at a rate of 2 km/sec. How long does it take for the area A to be at least 5 times its original size?

34. Computer Imaging New Age Special Effects, Inc., prepares computer software based on specifications prepared by film directors. To simulate an approaching vehicle, they begin with a computer image of a 5-cm by 7-cm by 3-cm box. The program increases each dimension at a rate of 2 cm/sec. How long does it take for the volume V of the box to be at least 5 times its initial size?

35. Which of the ordered pairs $(1, 1)$, $(4, -2)$, and $(3, -1)$ are in the relation given by $3x + 4y = 5$?

36. Which of the ordered pairs $(5, 1)$, $(3, 4)$, and $(0, -5)$ are in the relation given by $x^2 + y^2 = 25$?

In Exercises 37–44, find two functions defined implicitly by the given relation.

37. $x^2 + y^2 = 25$

38. $x + y^2 = 25$

39. $x^2 - y^2 = 25$

40. $3x^2 - y^2 = 25$

41. $x + |y| = 1$

42. $x - |y| = 1$

43. $y^2 = x^2$

44. $y^2 = x$

Standardized Test Questions

45. True or False The domain of the quotient function $(f/g)(x)$ consists of all numbers that belong to both the domain of f and the domain of g. Justify your answer.

46. True or False The domain of the product function $(fg)(x)$ consists of all numbers that belong to either the domain of f or the domain of g. Justify your answer.

You may use a graphing calculator when solving Exercises 47–50.

47. Multiple Choice Suppose f and g are functions with domain all real numbers. Which of the following statements is *not* necessarily true?

(A) $(f + g)(x) = (g + f)(x)$ **(B)** $(fg)(x) = (gf)(x)$

(C) $f(g(x)) = g(f(x))$ **(D)** $(f - g)(x) = -(g - f)(x)$

(E) $(f \circ g)(x) = f(g(x))$

48. Multiple Choice If $f(x) = x - 7$ and $g(x) = \sqrt{4 - x}$, what is the domain of the function f/g?

(A) $(-\infty, 4)$ **(B)** $(-\infty, 4]$ **(C)** $(4, \infty)$

(D) $[4, \infty)$ **(E)** $(4, 7) \cup (7, \infty)$

49. Multiple Choice If $f(x) = x^2 + 1$, then $(f \circ f)(x) =$

(A) $2x^2 + 2$ **(B)** $2x^2 + 1$ **(C)** $x^4 + 1$

(D) $x^4 + 2x^2 + 1$ **(E)** $x^4 + 2x^2 + 2$

50. Multiple Choice Which of the following relations defines the function $y = |x|$ implicitly?

(A) $y = x$ **(B)** $y^2 = x^2$ **(C)** $y^3 = x^3$

(D) $x^2 + y^2 = 1$ **(E)** $x = |y|$

Explorations

51. Three on a Match Match each function f with a function g and a domain D so that $(f \circ g)(x) = x^2$ with domain D.

f	g	D
e^x	$\sqrt{2 - x}$	$x \neq 0$
$(x^2 + 2)^2$	$x + 1$	$x \neq 1$
$(x^2 - 2)^2$	$2 \ln x$	$(0, \infty)$
$\dfrac{1}{(x - 1)^2}$	$\dfrac{1}{x - 1}$	$[2, \infty)$
$x^2 - 2x + 1$	$\sqrt{x - 2}$	$(-\infty, 2]$
$\left(\dfrac{x + 1}{x}\right)^2$	$\dfrac{x + 1}{x}$	$(-\infty, \infty)$

52. Be a *g* Whiz Let $f(x) = x^2 + 1$. Find a function g so that

(a) $(fg)(x) = x^4 - 1$

(b) $(f + g)(x) = 3x^2$

(c) $(f/g)(x) = 1$

(d) $f(g(x)) = 9x^4 + 1$

(e) $g(f(x)) = 9x^4 + 1$

Extending the Ideas

53. Identifying Identities An *identity* for a function operation is a function that combines with a given function f to return the same function f. Find the identity functions for the following operations:

(a) Function addition. That is, find a function g such that $(f + g)(x) = (g + f)(x) = f(x)$.

(b) Function multiplication. That is, find a function g such that $(fg)(x) = (gf)(x) = f(x)$.

(c) Function composition. That is, find a function g such that $(f \circ g)(x) = (g \circ f)(x) = f(x)$.

54. Is Function Composition Associative? You already know that function composition is not commutative; that is, $(f \circ g)(x) \neq (g \circ f)(x)$. But is function composition associative? That is, does $(f \circ (g \circ h))(x) = ((f \circ g) \circ h))(x)$? Explain your answer.

55. Revisiting Example 6 Solve $x^2y + y^2 = 5$ for y using the quadratic formula and graph the pair of implicit functions.