1.5 Parametric Relations and Inverses

What you'll learn about

- Relations Defined Parametrically
- Inverse Relations and Inverse Functions

... and why

Some functions and graphs can best be defined parametrically, while some others can be best understood as inverses of functions we already know.

Relations Defined Parametrically

Another natural way to define functions or, more generally, relations, is to define *both* elements of the ordered pair (x, y) in terms of another variable t, called a **parameter**. We illustrate with an example.

EXAMPLE 1 Defining a Function Parametrically

Consider the set of all ordered pairs (x, y) defined by the equations

$$x = t + 1$$
$$y = t^2 + 2t$$

where t is any real number.

(a) Find the points determined by $t = -3, -2, -1, 0, 1, 2,$ and 3.

(b) Find an algebraic relationship between x and y. (This is often called "eliminating the parameter.") Is y a function of x?

(c) Graph the relation in the (x, y) plane.

SOLUTION

(a) Substitute each value of t into the formulas for x and y to find the point that it determines parametrically:

t	$x = t + 1$	$y = t^2 + 2t$	(x, y)
-3	-2	3	$(-2, 3)$
-2	-1	0	$(-1, 0)$
-1	0	-1	$(0, -1)$
0	1	0	$(1, 0)$
1	2	3	$(2, 3)$
2	3	8	$(3, 8)$
3	4	15	$(4, 15)$

(b) We can find the relationship between x and y algebraically by the method of substitution. First solve for t in terms of x to obtain $t = x - 1$.

$$y = t^2 + 2t \qquad \text{Given}$$
$$y = (x - 1)^2 + 2(x - 1) \qquad t = x - 1$$
$$= x^2 - 2x + 1 + 2x - 2 \qquad \text{Expand.}$$
$$= x^2 - 1 \qquad \text{Simplify.}$$

This is consistent with the ordered pairs we had found in the table. As t varies over all real numbers, we will get all the ordered pairs in the relation $y = x^2 - 1$, which does indeed define y as a function of x.

(c) Since the parametrically defined relation consists of all ordered pairs in the relation $y = x^2 - 1$, we can get the graph by simply graphing the parabola $y = x^2 - 1$. See Figure 1.62. *Now try Exercise 5.*

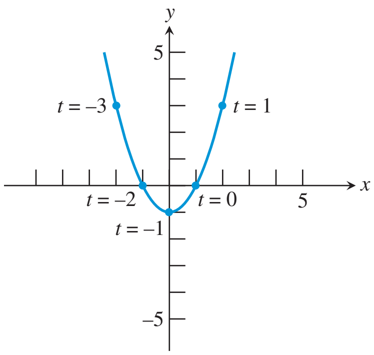


FIGURE 1.62 (Example 1)

EXAMPLE 2 Using a Graphing Calculator in Parametric Mode

Consider the set of all ordered pairs (x, y) defined by the equations

$$x = t^2 + 2t$$
$$y = t + 1$$

where t is any real number.

(a) Use a graphing calculator to find the points determined by $t = -3, 2, -1, 0, 1, 2,$ and 3.

(b) Use a graphing calculator to graph the relation in the (x, y) plane.

(c) Is y a function of x?

(d) Find an algebraic relationship between x and y.

t	(x, y)
-3	$(3, -2)$
-2	$(0, -1)$
-1	$(-1, 0)$
0	$(0, 1)$
1	$(3, 2)$
2	$(8, 3)$
3	$(15, 4)$

SOLUTION

(a) When the calculator is in *parametric mode*, the "Y =" screen provides a space to enter both X and Y as functions of the parameter T (Figure 1.63a). After entering the functions, use the table setup in Figure 1.63b to obtain the table shown in Figure 1.63c. The table shows, for example, that when $T = -3$ we have $X1T = 3$ and $Y1T = -2$, so the ordered pair corresponding to $t = -3$ is $(3, -2)$.

(b) In parametric mode, the "WINDOW" screen contains the usual x-axis information, as well as "Tmin," "Tmax," and "Tstep" (Figure 1.64a). To include most of the points listed in part (a), we set $\text{Xmin} = -5$, $\text{Xmax} = 5$, $\text{Ymin} = -3$, and $\text{Ymax} = 3$. Since $t = y - 1$, we set Tmin and Tmax to values one less than those for Ymin and Ymax.

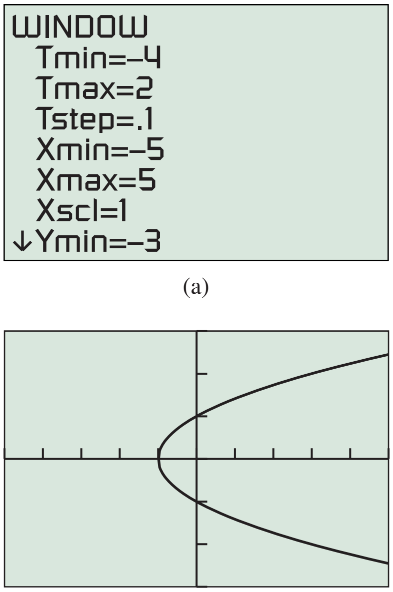
The value of Tstep determines how far the grapher will go from one value of t to the next as it computes the ordered pairs. With $\text{Tmax} - \text{Tmin} = 6$ and $\text{Tstep} = 0.1$, the grapher will compute 60 points, which is sufficient. (The more points, the smoother the graph. See Exploration 1.) The graph is shown in Figure 1.64b. Use TRACE to find some of the points found in (a).

(c) No, y is not a function of x. We can see this from part (a) because $(0, -1)$ and $(0, 1)$ have the same x-value but different y-values. Alternatively, notice that the graph in (b) fails the vertical line test.

(d) We can use the same algebraic steps as in Example 1 to get the relation in terms of x and y: $x = y^2 - 1$. ***Now try Exercise 7.***

(a) (b) (c)

FIGURE 1.63 Using the table feature of a grapher set in parametric mode. (Example 2)



(a)

[–5, 5] by [–3, 3]

(b)

FIGURE 1.64 The graph of a parabola in parametric mode on a graphing calculator. (Example 2)

EXPLORATION 1 Watching your Tstep

1. Graph the parabola in Example 2 in parametric mode as described in the solution. Press TRACE and observe the values of T, X, and Y. At what value of T does the calculator begin tracing? What point on the parabola results? (It's off the screen.) At what value of T does it stop tracing? What point on the parabola results? How many points are computed as you TRACE from start to finish?
2. Leave everything else the same and change the Tstep to 0.01. Do you get a smoother graph? Why or why not?
3. Leave everything else the same and change the Tstep to 1. Do you get a smoother graph? Why or why not?
4. What effect does the Tstep have on the speed of the grapher? Is this easily explained?
5. Now change the Tstep to 2. Why does the left portion of the parabola disappear? (It may help to TRACE along the curve.)
6. Change the Tstep back to 0.1 and change the Tmin to -1. Why does the bottom side of the parabola disappear? (Again, it may help to TRACE.)
7. Make a change to the window that will cause the grapher to show the bottom side of the parabola but not the top.

Time For T

Functions defined by parametric equations are frequently encountered in problems of motion, where the x- and y-coordinates of a moving object are computed as functions of time. This makes time the parameter, which is why you almost always see parameters given as "t" in parametric equations.

Inverse Relations and Inverse Functions

What happens when we reverse the coordinates of all the ordered pairs in a relation? We obviously get another relation, as it is another set of ordered pairs, but does it bear any resemblance to the original relation? If the original relation happens to be a function, will the new relation also be a function?

We can get some idea of what happens by examining Examples 1 and 2. The ordered pairs in Example 2 can be obtained by simply reversing the coordinates of the ordered pairs in Example 1. This is because we set up Example 2 by switching the parametric equations for x and y that we used in Example 1. We say that the relation in Example 2 is the *inverse relation* of the relation in Example 1.

DEFINITION Inverse Relation

The ordered pair (a, b) is in a relation if and only if the ordered pair (b, a) is in the **inverse relation**.

We will study the connection between a relation and its inverse. We will be most interested in inverse relations that happen to be *functions*. Notice that the graph of the inverse relation in Example 2 fails the vertical line test and is therefore not the graph of a function. Can we predict this failure by considering the graph of the original relation in Example 1? Figure 1.65 suggests that we can.

The inverse graph in Figure 1.65b fails the vertical line test because two different y-values have been paired with the same x-value. This is a direct consequence of the fact that the original relation in Figure 1.65a paired two different x-values with the same y-value. The inverse graph fails the *vertical* line test precisely because the original graph fails the *horizontal* line test. This gives us a test for relations whose inverses are functions.

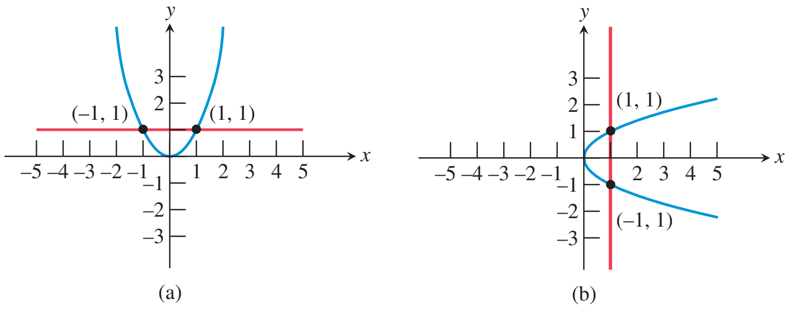


FIGURE 1.65 The inverse relation in (b) fails the vertical line test because the original relation in (a) fails the horizontal line test.

Horizontal Line Test

The inverse of a relation is a function if and only if each horizontal line intersects the graph of the original relation in at most one point.

EXAMPLE 3 Applying the Horizontal Line Test

Which of the graphs (1)–(4) in Figure 1.66 are graphs of

(a) relations that are functions?

(b) relations that have inverses that are functions?

SOLUTION

(a) Graphs (1) and (4) are graphs of functions because these graphs pass the vertical line test. Graphs (2) and (3) are not graphs of functions because these graphs fail the vertical line test.

(b) Graphs (1) and (2) are graphs of relations whose inverses are functions because these graphs pass the horizontal line test. Graphs (3) and (4) fail the horizontal line test so their inverse relations are not functions. ***Now try Exercise 9.***

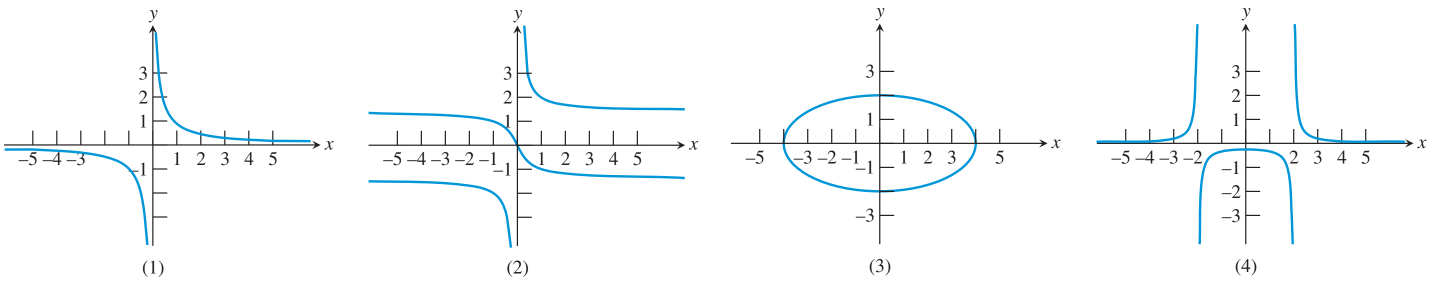


FIGURE 1.66 (Example 3)

A *function* whose inverse is a function has a graph that passes both the horizontal and vertical line tests (such as graph (1) in Example 3). Such a function is **one-to-one**, since every x is paired with a unique y and every y is paired with a unique x.

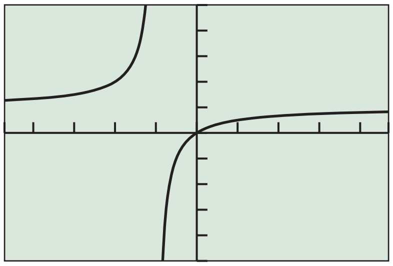
Caution about Function Notation

The symbol f^{-1} is read "f inverse" and should never be confused with the reciprocal of f. If f is a function, the symbol f^{-1}, can *only* mean f inverse. The reciprocal of f must be written as $1/f$.

DEFINITION Inverse Function

If f is a one-to-one function with domain D and range R, then the **inverse function of f**, denoted f^{-1}, is the function with domain R and range D defined by

$$f^{-1}(b) = a \quad \text{if and only if} \quad f(a) = b.$$



[–4.7, 4.7] by [–5, 5]

FIGURE 1.67 The graph of $f(x) = x/(x + 1)$. (Example 4)

EXAMPLE 4 Finding an Inverse Function Algebraically

Find an equation for $f^{-1}(x)$ if $f(x) = x/(x + 1)$.

SOLUTION The graph of f in Figure 1.67 suggests that f is one-to-one. The original function satisfies the equation $y = x/(x + 1)$. If f truly is one-to-one, the inverse function f^{-1} will satisfy the equation $x = y/(y + 1)$. (Note that we just switch the x and the y.)

If we solve this new equation for y we will have a formula for $f^{-1}(x)$:

$$x = \frac{y}{y + 1}$$

$$x(y + 1) = y \quad \text{Multiply by } y + 1.$$

$$xy + x = y \quad \text{Distributive property}$$

$$xy - y = -x \quad \text{Isolate the } y \text{ terms.}$$

$$y(x - 1) = -x \quad \text{Factor out } y.$$

$$y = \frac{-x}{x - 1} \quad \text{Divide by } x - 1.$$

$$y = \frac{x}{1 - x} \quad \text{Multiply numerator and denominator by } -1.$$

Therefore $f^{-1}(x) = x/(1 - x)$. *Now try Exercise 15.*

Let us candidly admit two things regarding Example 4 before moving on to a graphical model for finding inverses. First, many functions are not one-to-one and so do not have inverse functions. Second, the algebra involved in finding an inverse function in the manner of Example 4 can be extremely difficult. We will actually find very few inverses this way. As you will learn in future chapters, we will usually rely on our understanding of how f maps x to y to understand how f^{-1} maps y to x.

It is possible to use the graph of f to produce a graph of f^{-1} without doing any algebra at all, thanks to the following geometric reflection property:

The Inverse Reflection Principle

The points (a, b) and (b, a) in the coordinate plane are symmetric with respect to the line $y = x$. The points (a, b) and (b, a) are **reflections** of each other across the line $y = x$.

EXAMPLE 5 Finding an Inverse Function Graphically

The graph of a function $y = f(x)$ is shown in Figure 1.68. Sketch a graph of the function $y = f^{-1}(x)$. Is f a one-to-one function?

SOLUTION We need not find a formula for $f^{-1}(x)$. All we need to do is to find the reflection of the given graph across the line $y = x$. This can be done geometrically.

Imagine a mirror along the line $y = x$ and draw the reflection of the given graph in the mirror (Figure 1.69).

Another way to visualize this process is to imagine the graph to be drawn on a large pane of glass. Imagine the glass rotating around the line $y = x$ so that the *positive* x-axis switches places with the *positive* y-axis. (The back of the glass must be rotated to the front for this to occur.) The graph of f will then become the graph of f^{-1}.

Since the inverse of f has a graph that passes the horizontal and vertical line test, f is a one-to-one function. *Now try Exercise 23.*

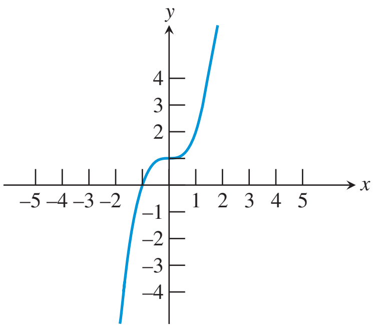
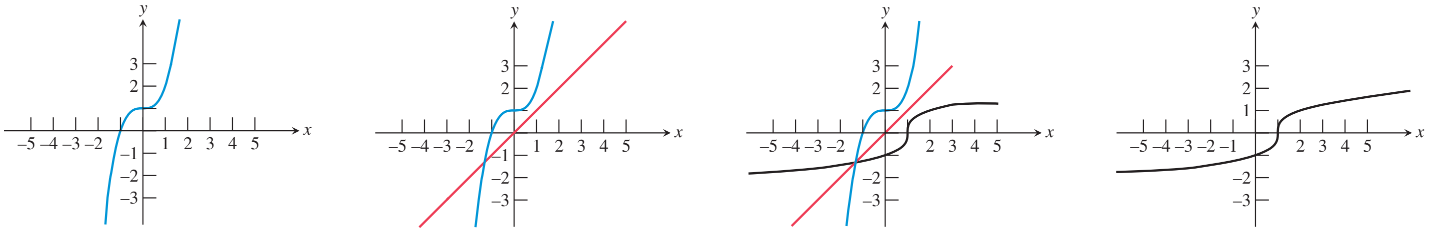


FIGURE 1.68 The graph of a one-to-one function. (Example 5)



The graph of f. The mirror $y = x$. The reflection. The graph of f^{-1}.

FIGURE 1.69 The mirror method. The graph of f is reflected in an imaginary mirror along the line $y = x$ to produce the graph of f^{-1}. (Example 5)

There is a natural connection between inverses and function composition that gives further insight into what an inverse actually does: It "undoes" the action of the original function. This leads to the following rule:

The Inverse Composition Rule

A function f is one-to-one with inverse function g if and only if

$$f(g(x)) = x \text{ for every } x \text{ in the domain of } g, \text{ and}$$
$$g(f(x)) = x \text{ for every } x \text{ in the domain of } f.$$

EXAMPLE 6 Verifying Inverse Functions

Show algebraically that $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x - 1}$ are inverse functions.

SOLUTION We use the Inverse Composition Rule.

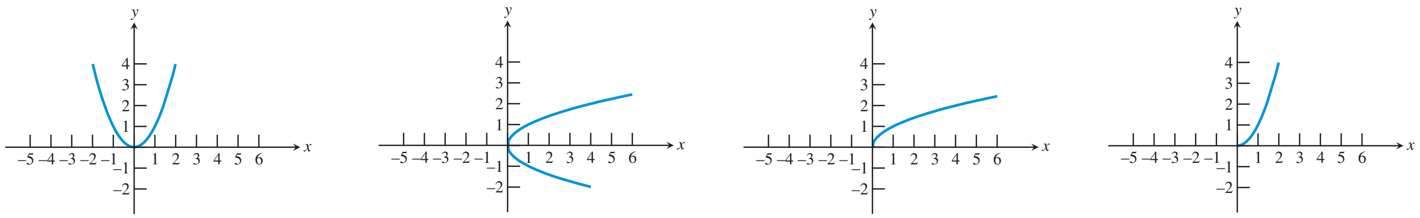
$$f(g(x)) = f(\sqrt[3]{x-1}) = (\sqrt[3]{x-1})^3 + 1 = x - 1 + 1 = x$$
$$g(f(x)) = g(x^3 + 1) = \sqrt[3]{(x^3+1) - 1} = \sqrt[3]{x^3} = x$$

Since these equations are true for all x, the Inverse Composition Rule guarantees that f and g are inverses.

You do not have far to go to find graphical support of this algebraic verification, since these are the functions whose graphs are shown in Example 5!

Now try Exercise 27.

Some functions are so important that we need to study their inverses even though they are not one-to-one. A good example is the square root function, which is the "inverse" of the square function. It is not the inverse of the *entire* squaring function, because the full parabola fails the horizontal line test. Figure 1.70 shows that the function $y = \sqrt{x}$ is really the inverse of a "restricted-domain" version of $y = x^2$ defined only for $x \geq 0$.



The graph of $y = x^2$ (not one-to-one). The inverse relation of $y = x^2$ (not a function). The graph of $y = \sqrt{x}$ (a function). The graph of the function whose inverse is $y = \sqrt{x}$.

FIGURE 1.70 The function $y = x^2$ has no inverse function, but $y = \sqrt{x}$ is the inverse function of $y = x^2$ on the restricted domain $[0, \infty)$.

The consideration of domains adds a refinement to the algebraic inverse-finding method of Example 4, which we now summarize:

How to Find an Inverse Function Algebraically

Given a formula for a function f, proceed as follows to find a formula for f^{-1}.

1. Determine that there is a function f^{-1} by checking that f is one-to-one. State any restrictions on the domain of f. (Note that it might be necessary to impose some to get a one-to-one version of f.)
2. Switch x and y in the formula $y = f(x)$.
3. Solve for y to get the formula $y = f^{-1}(x)$. State any restrictions on the domain of f^{-1}.

EXAMPLE 7 Finding an Inverse Function

Show that $f(x) = \sqrt{x + 3}$ has an inverse function and find a rule for $f^{-1}(x)$. State any restrictions on the domains of f and f^{-1}.

SOLUTION

Solve Algebraically

The graph of f passes the horizontal line test, so f has an inverse function (Figure 1.71). Note that f has domain $[-3, \infty)$ and range $[0, \infty)$.

To find f^{-1} we write

$y = \sqrt{x + 3}$ where $x \geq -3, y \geq 0$

$x = \sqrt{y + 3}$ where $y \geq -3, x \geq 0$ Interchange x and y.

$x^2 = y + 3$ where $y \geq -3, x \geq 0$ Square.

$y = x^2 - 3$ where $y \geq -3, x \geq 0$ Solve for y.

Thus $f^{-1}(x) = x^2 - 3$, with an "inherited" domain restriction of $x \geq 0$. Figure 1.71 shows the two functions. Note the domain restriction of $x \geq 0$ imposed on the parabola $y = x^2 - 3$.

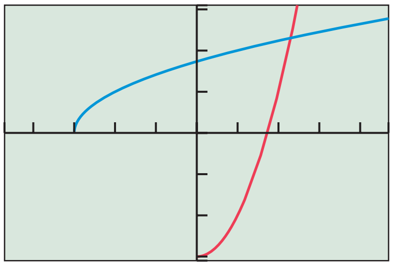
Support Graphically

Use a grapher in parametric mode and compare the graphs of the two sets of parametric equations with Figure 1.71:

$x = t$ and $x = \sqrt{t + 3}$

$y = \sqrt{t + 3}$ $y = t$

Now try Exercise 17.



[–4.7, 4.7] by [–3.1, 3.1]

FIGURE 1.71 The graph of $f(x) = \sqrt{x + 3}$ and its inverse, a restricted version of $y = x^2 - 3$. (Example 7)

QUICK REVIEW 1.5 *(For help, go to Section P.3 and P.4.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–10, solve the equation for y.

1. $x = 3y - 6$
2. $x = 0.5y + 1$
3. $x = y^2 + 4$
4. $x = y^2 - 6$
5. $x = \dfrac{y - 2}{y + 3}$
6. $x = \dfrac{3y - 1}{y + 2}$
7. $x = \dfrac{2y + 1}{y - 4}$
8. $x = \dfrac{4y + 3}{3y - 1}$
9. $x = \sqrt{y + 3}, y \geq -3$
10. $x = \sqrt{y - 2}, y \geq 2$

SECTION 1.5 EXERCISES

In Exercises 1–4, find the (x, y) pair for the value of the parameter.

1. $x = 3t$ and $y = t^2 + 5$ for $t = 2$

2. $x = 5t - 7$ and $y = 17 - 3t$ for $t = -2$

3. $x = t^3 - 4t$ and $y = \sqrt{t + 1}$ for $t = 3$

4. $x = |t + 3|$ and $y = 1/t$ for $t = -8$

In Exercises 5–8, complete the following. **(a)** Find the points determined by $t = -3, -2, -1, 0, 1, 2,$ and 3. **(b)** Find a direct algebraic relationship between x and y and determine whether the parametric equations determine y as a function of x. **(c)** Graph the relationship in the xy-plane.

5. $x = 2t$ and $y = 3t - 1$

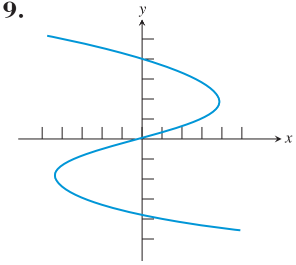
6. $x = t + 1$ and $y = t^2 - 2t$

7. $x = t^2$ and $y = t - 2$

8. $x = \sqrt{t}$ and $y = 2t - 5$

In Exercises 9–12, the graph of a relation is shown. **(a)** Is the relation a function? **(b)** Does the relation have an inverse that is a function?

9.



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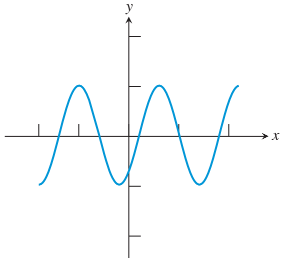

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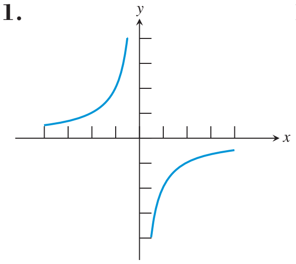

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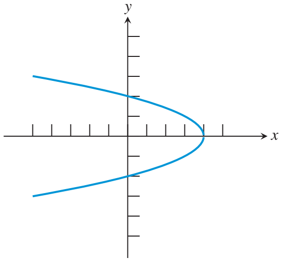


In Exercises 13–22, find a formula for $f^{-1}(x)$. Give the domain of f^{-1}, including any restrictions "inherited" from f.

13. $f(x) = 3x - 6$

14. $f(x) = 2x + 5$

15. $f(x) = \dfrac{2x - 3}{x + 1}$

16. $f(x) = \dfrac{x + 3}{x - 2}$

17. $f(x) = \sqrt{x - 3}$

18. $f(x) = \sqrt{x + 2}$

19. $f(x) = x^3$

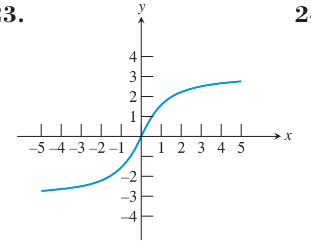
20. $f(x) = \sqrt{x^3 + 5}$

21. $f(x) = \sqrt[3]{x + 5}$

22. $f(x) = \sqrt[3]{x - 2}$

In Exercises 23–26, determine whether the function is one-to-one. If it is one-to-one, sketch the graph of the inverse.

23.



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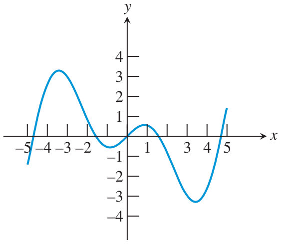

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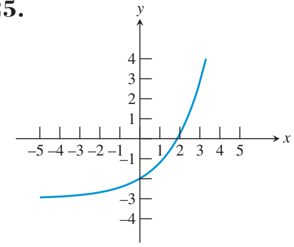

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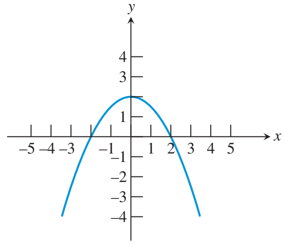

In Exercises 27–32, confirm that f and g are inverses by showing that $f(g(x)) = x$ and $g(f(x)) = x$.

27. $f(x) = 3x - 2$ and $g(x) = \dfrac{x + 2}{3}$

28. $f(x) = \dfrac{x + 3}{4}$ and $g(x) = 4x - 3$

29. $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x - 1}$

30. $f(x) = \dfrac{7}{x}$ and $g(x) = \dfrac{7}{x}$

31. $f(x) = \dfrac{x + 1}{x}$ and $g(x) = \dfrac{1}{x - 1}$

32. $f(x) = \dfrac{x + 3}{x - 2}$ and $g(x) = \dfrac{2x + 3}{x - 1}$

33. Currency Conversion In May of 2002 the exchange rate for converting U.S. dollars (x) to euros (y) was $y = 1.08x$.

(a) How many euros could you get for \$100 U.S.?

(b) What is the inverse function, and what conversion does it represent?

(c) In the spring of 2002, a tourist had an elegant lunch in Provence, France, ordering from a "fixed price" 48-euro menu. How much was that in U.S. dollars?

34. Temperature Conversion The formula for converting Celsius temperature (x) to Kelvin temperature is $k(x) = x + 273.16$. The formula for converting Fahrenheit temperature (x) to Celsius temperature is $c(x) = (5/9)(x - 32)$.

(a) Find a formula for $c^{-1}(x)$. What is this formula used for?

(b) Find $(k \circ c)(x)$. What is this formula used for?

35. Which pairs of basic functions (Section 1.3) are inverses of each other?

36. Which basic functions (Section 1.3) are their own inverses?

37. Which basic function can be defined parametrically as follows?

$$x = t^3 \text{ and } y = \sqrt{t^6} \text{ for } -\infty < t < \infty$$

38. Which basic function can be defined parametrically as follows?

$$x = 8t^3 \text{ and } y = (2t)^3 \text{ for } -\infty < t < \infty$$

Standardized Test Questions

39. True or False If f is a one-to-one function with domain D and range R, then f^{-1} is a one-to-one function with domain R and range D. Justify your answer.

40. True or False The set of points $(t + 1, 2t + 3)$ for all real numbers t form a line with slope 2. Justify your answer.

In Exercises 41–44, answer the questions without using a calculator.

41. Multiple Choice Which ordered pair is in the *inverse* of the relation given by $x^2y + 5y = 9$?

(A) $(2, 1)$ **(B)** $(-2, 1)$ **(C)** $(-1, 2)$ **(D)** $(2, -1)$

(E) $(1, -2)$

42. Multiple Choice Which ordered pair is not in the *inverse* of the relation given by $xy^2 - 3x = 12$?

(A) $(0, -4)$ **(B)** $(4, 1)$ **(C)** $(3, 2)$ **(D)** $(2, 12)$

(E) $(1, -6)$

43. Multiple Choice Which function is the *inverse* of the function $f(x) = 3x - 2$?

(A) $g(x) = \dfrac{x}{3} + 2$

(B) $g(x) = 2 - 3x$

(C) $g(x) = \dfrac{x + 2}{3}$

(D) $g(x) = \dfrac{x - 3}{2}$

(E) $g(x) = \dfrac{x - 2}{3}$

44. Multiple Choice Which function is the *inverse* of the function $f(x) = x^3 + 1$?

(A) $g(x) = \sqrt[3]{x - 1}$

(B) $g(x) = \sqrt[3]{x} - 1$

(C) $g(x) = x^3 - 1$

(D) $g(x) = \sqrt[3]{x + 1}$

(E) $g(x) = 1 - x^3$

Explorations

45. Function Properties Inherited by Inverses There are some properties of functions that are automatically shared by inverse functions (when they exist) and some that are not. Suppose that f has an inverse function f^{-1}. Give an algebraic or graphical argument (not a rigorous formal proof) to show that each of these properties of f must necessarily be shared by f^{-1}.

(a) f is continuous.

(b) f is one-to-one.

(c) f is odd (graphically, symmetric with respect to the origin).

(d) f is increasing.

46. Function Properties Not Inherited by Inverses There are some properties of functions that are not necessarily shared by inverse functions, even if the inverses exist. Suppose that f has an inverse function f^{-1}. For each of the following properties, give an example to show that f can have the property while f^{-1} does not.

(a) f has a graph with a horizontal asymptote.

(b) f has domain all real numbers.

(c) f has a graph that is bounded above.

(d) f has a removable discontinuity at $x = 5$.

47. Scaling Algebra Grades A teacher gives a challenging algebra test to her class. The lowest score is 52, which she decides to scale to 70. The highest score is 88, which she decides to scale to 97.

(a) Using the points $(52, 70)$ and $(88, 97)$, find a linear equation that can be used to convert raw scores to scaled grades.

(b) Find the inverse of the function defined by this linear equation. What does the inverse function do?

48. Writing to Learn (Continuation of Exercise 47) Explain why it is important for fairness that the scaling function used by the teacher be an *increasing* function. (*Caution:* It is *not* because "everyone's grade must go up." What would the scaling function in Exercise 47 do for a student who does enough "extra credit" problems to get a raw score of 136?)

Extending the Ideas

49. Modeling a Fly Ball Parametrically A baseball that leaves the bat at an angle of 60° from horizontal traveling 110 feet per second follows a path that can be modeled by the following pair of parametric equations. (You might enjoy verifying this if you have studied motion in physics.)

$$x = 110(t)\cos(60°)$$
$$y = 110(t)\sin(60°) - 16t^2$$

You can simulate the flight of the ball on a grapher. Set your grapher to parametric mode and put the functions above in for X2T and Y2T. Set X1T = 325 and Y1T = 5T to draw a 30-foot fence 325 feet from home plate. Set Tmin = 0, Tmax = 6, Tstep = 0.1, Xmin = 0, Xmax = 350, Xscl = 0, Ymin = 0, Ymax = 300, and Yscl = 0.

(a) Now graph the function. Does the fly ball clear the fence?

(b) Change the angle to 30° and run the simulation again. Does the ball clear the fence?

(c) What angle is optimal for hitting the ball? Does it clear the fence when hit at that angle?

50. The Baylor GPA Scale Revisited *(See Problem 78 in Section 1.2.)* The function used to convert Baylor School percentage grades to GPAs on a 4-point scale is

$$y = \left(\frac{3^{1.7}}{30}(x - 65)\right)^{\frac{1}{1.7}} + 1.$$

The function has domain [65, 100]. Anything below 65 is a failure and automatically converts to a GPA of 0.

(a) Find the inverse function algebraically. What can the inverse function be used for?

(b) Does the inverse function have any domain restrictions?

(c) Verify with a graphing calculator that the function found in (a) and the given function are really inverses.

51. Group Activity (Continuation of Exercise 50) The number 1.7 that appears in two places in the GPA scaling formula is called the scaling factor (k). The value of k can be changed to alter the curvature of the graph while keeping the points $(65, 1)$ and $(95, 4)$ fixed. It was felt that the lowest D (65) needed to be scaled to 1.0, while the middle A (95) needed to be scaled to 4.0. The faculty's Academic Council considered several values of k before settling on 1.7 as the number that gives the "fairest" GPAs for the other percentage grades.

Try changing k to other values between 1 and 2. What kind of scaling curve do you get when $k = 1$? Do you agree with the Baylor decision that $k = 1.7$ gives the fairest GPAs?