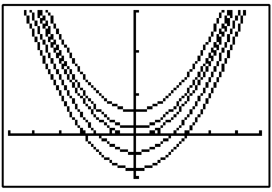
■ Section 1.6 Graphical Transformations

Exploration 1

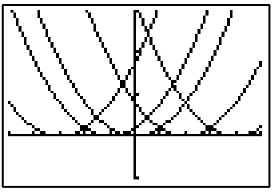
1.



[−5, 5] by [−5, 15]

They raise or lower the parabola along the y-axis.

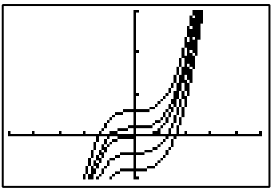
2.



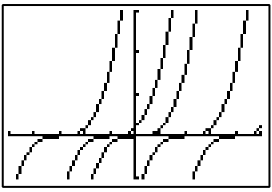
[−5, 5] by [−5, 15]

They move the parabola left or right along the x-axis.

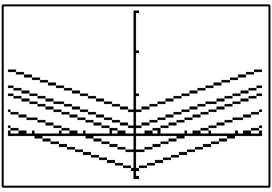
3.



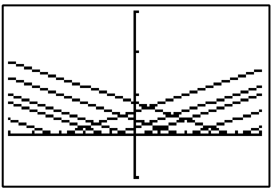
[−5, 5] by [−5, 15]



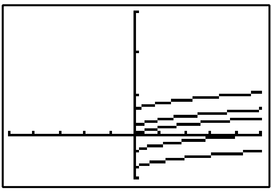
[−5, 5] by [−5, 15]



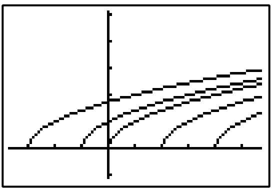
[−5, 5] by [−5, 15]



[−5, 5] by [−5, 15]



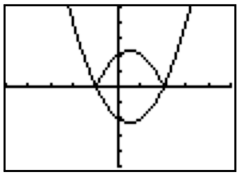
[−5, 5] by [−5, 15]



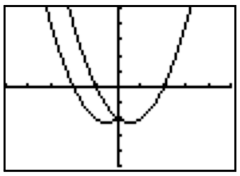
[−3.7, 5.7] by [−1.1, 5.1]

Yes

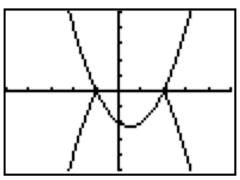
Exploration 2

1. 

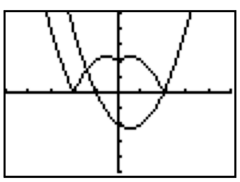
Graph C. Points with positive y-coordinates remain unchanged, while points with negative y-coordinates are reflected across the x-axis.

2. 

Graph A. Points with positive x-coordinates remain unchanged. Since the new function is even, the graph for negative x-values will be a reflection of the graph for positive x-values.

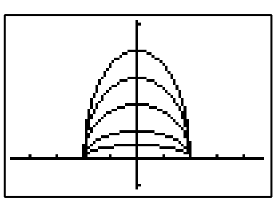
3. 

Graph F. The graph will be a reflection across the x-axis of graph C.

4. 

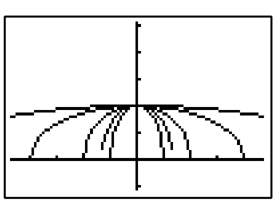
Graph D. The points with negative y-coordinates in graph A are reflected across the x-axis.

Exploration 3

1. 

$[-4.7, 4.7]$ by $[-1.1, 5.1]$

The 1.5 and the 2 stretch the graph vertically; the 0.5 and the 0.25 shrink the graph vertically.

2. 

$[-4.7, 4.7]$ by $[-1.1, 5.1]$

The 1.5 and the 2 shrink the graph horizontally; the 0.5 and the 0.25 stretch the graph horizontally.

Quick Review 1.6

1. $(x + 1)^2$
2. $(x - 3)^2$
3. $(x + 6)^2$
4. $(2x + 1)^2$
5. $(x - 5/2)^2$
6. $(2x - 5)^2$
7. $x^2 - 4x + 4 + 3x - 6 + 4 = x^2 - x + 2$
8. $2(x^2 + 6x + 9) - 5x - 15 - 2 =$ $2x^2 + 12x + 18 - 5x - 17 = 2x^2 + 7x + 1$
9. $(x^3 - 3x^2 + 3x - 1) + 3(x^2 - 2x + 1) - 3x + 3$ $= x^3 - 3x^2 + 2 + 3x^2 - 6x + 3 = x^3 - 6x + 5$
10. $2(x^3 + 3x^2 + 3x + 1) - 6(x^2 + 2x + 1) +$ $6x + 6 - 2 = 2x^3 + 6x^2 + 6x + 2 - 6x^2 - 12x$ $- 6 + 6x + 6 - 2 = 2x^3$

Section 1.6 Exercises

1. Vertical translation down 3 units
2. Vertical translation up 5.2 units
3. Horizontal translation left 4 units
4. Horizontal translation right 3 units
5. Horizontal translation to the right 100 units
6. Vertical translation down 100 units
7. Horizontal translation to the right 1 unit, and vertical translation up 3 units
8. Horizontal translation to the left 50 units and vertical translation down 279 units
9. Reflection across x-axis
10. Horizontal translation right 5 units
11. Reflection across y-axis
12. This can be written as $y = \sqrt{-(x - 3)}$ or $y = \sqrt{-x + 3}$. The first of these can be interpreted as reflection across the y-axis followed by a horizontal translation to the right 3 units. The second may be viewed as a horizontal translation left 3 units followed by a reflection across the y-axis.
    Note that when combining horizontal changes (horizontal translations and reflections across the y-axis), the order is "backwards" from what one may first expect: With $y = \sqrt{-(x - 3)}$, although we first subtract 3 from x then negate, the order of transformations is reflect then translate. With $y = \sqrt{-x + 3}$, although we negate x then add 3, the order of transformations is translate then reflect.

For #13–20, recognize $y = c \cdot x^3$ $(c > 0)$ as a vertical stretch (if $c > 1$) or shrink (if $0 < c < 1$) of factor c, and $y = (c \cdot x)^3$ as a horizontal shrink (if $c > 1$) or stretch (if $0 < c < 1$) of factor $1/c$. Note also that $y = (c \cdot x)^3 = c^3x^3$, so that for this function, any horizontal stretch/shrink can be interpreted as an equivalent vertical shrink/stretch (and vice versa).

13. Vertically stretch by 2
14. Horizontally shrink by 1/2, or vertically stretch by $2^3 = 8$
15. Horizontally stretch by $1/0.2 = 5$, or vertically shrink by $0.2^3 = 0.008$

16. Vertically shrink by 0.3

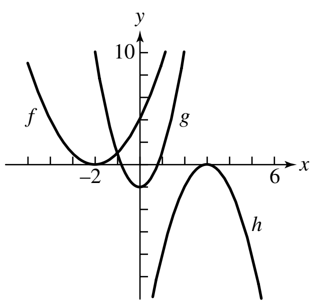
17. $g(x) = \sqrt{x - 6} + 2 = f(x - 6)$; starting with f, translate right 6 units to get g.

18. $g(x) = -(x + 4 - 1)^2 = -f(x + 4)$; starting with f, translate left 4 units, and reflect across the x-axis to get g.

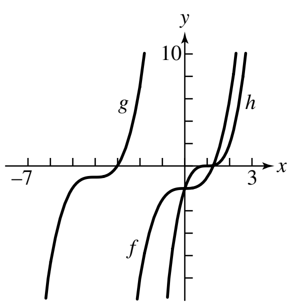
19. $g(x) = -(x + 4 - 2)^3 = -f(x + 4)$; starting with f, translate left 4 units, and reflect across the x-axis to get g.

20. $g(x) = 2|2x| = 2f(x)$; starting with f, vertically stretch by 2 to get g.

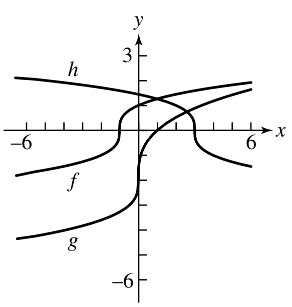
21.



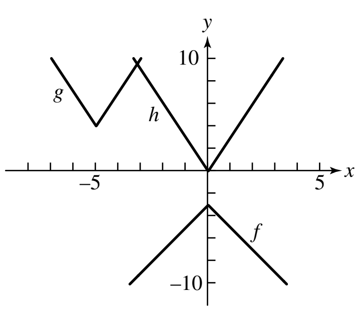
22.



23.



24.



25. Since the graph is translated left 5 units, $f(x) = \sqrt{x + 5}$.

26. The graph is reflected across the y-axis and translated right 3 units. $y = \sqrt{-x}$ would be reflected across the y-axis; the horizontal translation gives
$f(x) = \sqrt{-(x - 3)} = \sqrt{3 - x}$.
See also Exercise 12 in this section, and note accompanying that solution.

27. The graph is reflected across the x-axis, translated left 2 units, and translated up 3 units. $y = -\sqrt{x}$ would be reflected across the x-axis, $y = -\sqrt{x + 2}$ adds the horizontal translation, and finally, the vertical translation gives $f(x) = -\sqrt{x + 2} + 3 = 3 - \sqrt{x + 2}$.

28. The graph is vertically stretched by 2, translated left 5 units, and translated down 3 units. $y = 2\sqrt{x}$ would be vertically stretched, $y = 2\sqrt{x + 5}$ adds the horizontal translation, and finally, the vertical translation gives $f(x) = 2\sqrt{x + 5} - 3$.

29. **(a)** $y = -f(x) = -(x^3 - 5x^2 - 3x + 2)$
$= -x^3 + 5x^2 + 3x - 2$

(b) $y = f(-x) = (-x)^3 - 5(-x)^2 - 3(-x) + 2$
$= -x^3 - 5x^2 + 3x + 2$

30. **(a)** $y = -f(x) = -(2\sqrt{x + 3} - 4) = -2\sqrt{x + 3} + 4$

(b) $y = f(-x) = 2\sqrt{-x + 3} - 4 = 2\sqrt{3 - x} - 4$

31. **(a)** $y = -f(x) = -(\sqrt[3]{8x}) = -2\sqrt[3]{x}$

(b) $y = f(-x) = \sqrt[3]{8(-x)} = \sqrt[3]{-8x} = -2\sqrt[3]{x}$

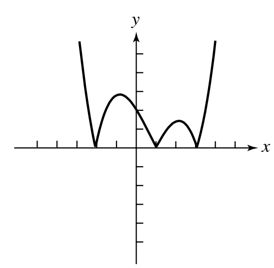
32. **(a)** $y = -f(x) = -3|x + 5|$

(b) $y = f(-x) = 3|-x + 5| = 3|5 - x|$

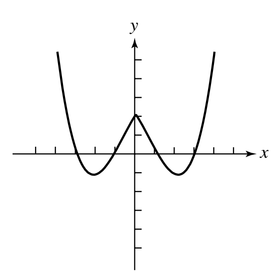
33. Let f be an odd function; that is, $f(-x) = -f(x)$ for all x in the domain of f. To reflect the graph of $y = f(x)$ across the y-axis, we make the transformation $y = f(-x)$. But $f(-x) = -f(x)$ for all x in the domain of f, so this transformation results in $y = -f(x)$. That is exactly the translation that reflects the graph of f across the x-axis, so the two reflections yield the same graph.

34. Let f be an odd function; that is, $f(-x) = -f(x)$ for all x in the domain of f. To reflect the graph of $y = f(x)$ across the y-axis, we make the transformation $y = f(-x)$. Then, reflecting across the x-axis yields $y = -f(-x)$. But $f(-x) = -f(x)$ for all x in the domain of f, so we have $y = -f(-x) = -[-f(x)] = f(x)$; that is, the original function.

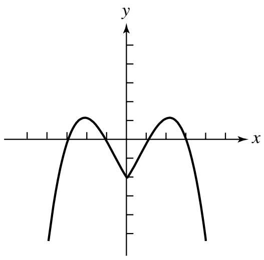
35.



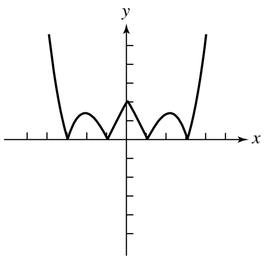
36.



37.



38.



39. **(a)** $y_1 = 2y = 2(x^3 - 4x) = 2x^3 - 8x$

(b) $y_2 = f\left(\frac{x}{\frac{1}{3}}\right) = f(3x) = (3x)^3 - 4(3x) = 27x^3 - 12x$

40. **(a)** $y_1 = 2y = 2|x + 2|$

(b) $y_2 = f(3x) = |3x + 2|$

41. **(a)** $y_1 = 2y = 2(x^2 + x - 2) = 2x^2 + 2x - 4$

(b) $y_2 = f(3x) = (3x)^2 + 3x - 2 = 9x^2 + 3x - 2$

42. **(a)** $y_1 = 2y = 2\left(\frac{1}{x + 2}\right) = \frac{2}{x + 2}$

(b) $y_2 = f(3x) = \frac{1}{3x + 2}$

43. Starting with $y = x^2$, translate right 3 units, vertically stretch by 2, and translate down 4 units.

44. Starting with $y = \sqrt{x}$, translate left 1 unit, vertically stretch by 3, and reflect across x-axis.

45. Starting with $y = x^2$, horizontally shrink by $\frac{1}{3}$ and translate down 4 units.

46. Starting with $y = |x|$, translate left 4 units, vertically stretch by 2, reflect across x-axis, and translate up 1 unit.

47. First stretch (multiply right side by 3): $y = 3x^2$, then translate (replace x with $x - 4$): $y = 3(x - 4)^2$.

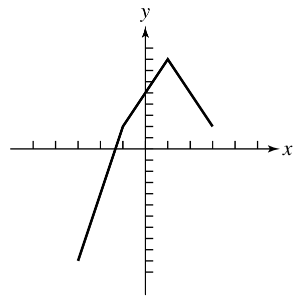
48. First translate (replace x with $x - 4$): $y = (x - 4)^2$, then stretch (multiply right side by 3): $y = 3(x - 4)^2$.

49. First translate left (replace x with $x + 2$): $y = |x + 2|$, then stretch (multiply right side by 2): $y = 2|x + 2|$, then translate down (subtract 4 from the right side): $y = 2|x + 2| - 4$.

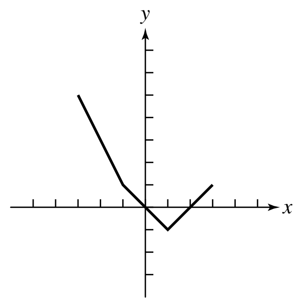
50. First translate left (replace x with $x + 2$): $y = |x + 2|$, then shrink (replace x with $2x$): $y = |2x + 2|$, then translate down (subtract 4 from the right side): $y = |2x + 2| - 4$. This can be simplified to $y = |2(x + 1)| - 4 = 2|x + 1| - 4$.

To make the sketches for #51–54, it is useful to apply the described transformations to several selected points on the graph. The original graph here has vertices $(-2, -4)$, $(0, 0)$, $(2, 2)$, and $(4, 0)$; in the solutions below, the images of these four points are listed.

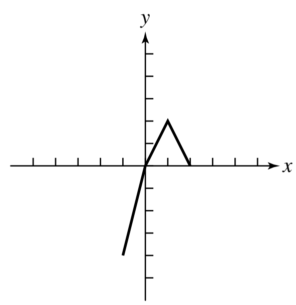
51. Translate left 1 unit, then vertically stretch by 3, and finally translate up 2 units. The four vertices are transformed to $(-3, -10)$, $(-1, 2)$, $(1, 8)$, and $(3, 2)$.



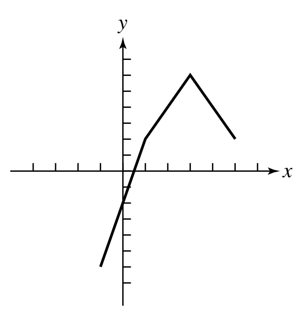
52. Translate left 1 unit, then reflect across the x-axis, and finally translate up 1 unit. The four vertices are transformed to $(-3, 5)$, $(-1, 1)$, $(1, -1)$, and $(3, 1)$.



53. Horizontally shrink by $\frac{1}{2}$. The four vertices are transformed to $(-1, -4)$, $(0, 0)$, $(1, 2)$, $(2, 0)$.



54. Translate right 1 unit, then vertically stretch by 2, and finally translate up 2 units. The four vertices are transformed to $(-1, -6)$, $(1, 2)$, $(3, 6)$, and $(5, 2)$.



55. Reflections have more effect on points that are farther away from the line of reflection. Translations affect the distance of points from the axes, and hence change the effect of the reflections.

56. The x-intercepts are the values at which the function equals zero. The stretching (or shrinking) factors have no effect on the number zero, so those y-coordinates do not change.

57. First vertically stretch by $\frac{9}{5}$, then translate up 32 units.

58. Solve for C: $F = \frac{9}{5}C + 32$, so $C = \frac{5}{9}(F - 32) = \frac{5}{9}F - \frac{160}{9}$. First vertically shrink by $\frac{5}{9}$, then translate down $\frac{160}{9} = 17.\bar{7}$ units.

59. False. $y = f(x + 3)$ is $y = f(x)$ translated 3 units to the *left.*

60. True. $y = f(x) - c$ represents a translation down by c units. (The translation is up when $c < 0$.)

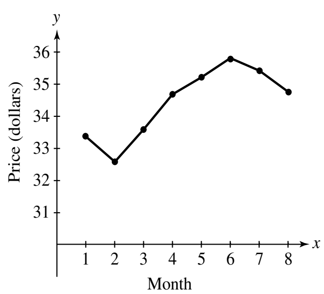
61. To vertically stretch $y = f(x)$ by a factor of 3, multiply the $f(x)$ by 3. The answer is C.

62. To translate $y = f(x)$ 4 units to the right, subtract 4 from x inside the $f(x)$. The answer is D.

63. To translate $y = f(x)$ 2 units up, add 2 to $f(x)$: $y = f(x) + 2$. To reflect the result across the y-axis, replace x with $-x$. The answer is A.

64. To reflect $y = f(x)$ across the x-axis, multiply $f(x)$ by -1: $y = -f(x)$. To shrink the result horizontally by a factor of $\frac{1}{2}$, replace x with $2x$. The answer is E.

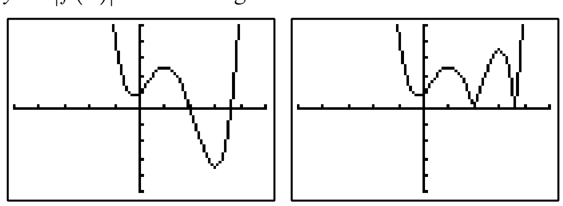
65. (a)



(b) Change the y-value by multiplying by the conversion rate from dollars to yen, a number that changes according to international market conditions. This results in a vertical stretch by the conversion rate.

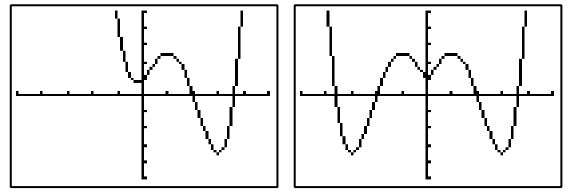
66. Apply the same transformation to the Ymin, Ymax, and Yscl as you apply to transform the function.

67. (a) The original graph is on the left; the graph of $y = |f(x)|$ is on the right.



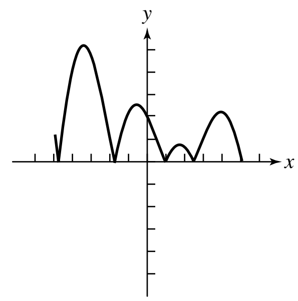
$[-5, 5]$ by $[-10, 10]$ $[-5, 5]$ by $[-10, 10]$

(b) The original graph is on the left; the graph of $y = f(|x|)$ is on the right.

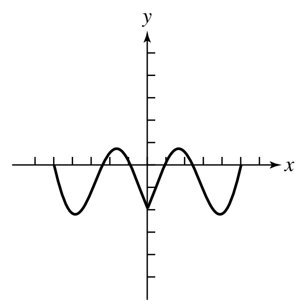


$[-5, 5]$ by $[-10, 10]$ $[-5, 5]$ by $[-10, 10]$

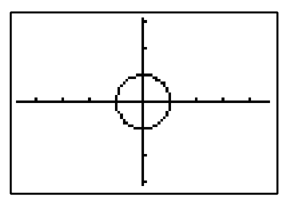
(c)



(d)

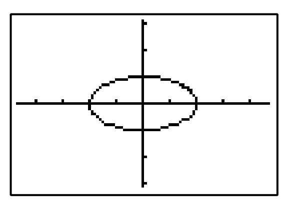


68. (a)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

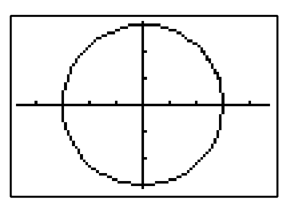
(b)



$x = 2 \cos t$
$y = \sin t$

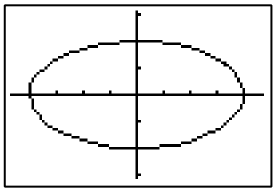
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(c)



$x = 3 \cos t$
$y = 3 \sin t$

$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(d) 

[−4.7, 4.7] by [−3.1, 3.1]

$x = 4 \cos t$
$y = 2 \sin t$