1.6 Graphical Transformations

What you'll learn about

- Transformations
- Vertical and Horizontal Translations
- Reflections Across Axes
- Vertical and Horizontal Stretches and Shrinks
- Combining Transformations

... and why

Studying transformations will help you to understand the relationships between graphs that have similarities but are not the same.

Transformations

The following functions are all different:

$$y = x^2$$
$$y = (x - 3)^2$$
$$y = 1 - x^2$$
$$y = x^2 - 4x + 5$$

However, a look at their graphs shows that, while no two are exactly the same, all four have the same identical *shape* and *size*. Understanding how algebraic alterations change the shapes, sizes, positions, and orientations of graphs is helpful for understanding the connection between algebraic and graphical models of functions.

In this section we relate graphs using **transformations**, which are functions that map real numbers to real numbers. By acting on the x-coordinates and y-coordinates of points, transformations change graphs in predictable ways. **Rigid transformations**, which leave the size and shape of a graph unchanged, include horizontal translations, vertical translations, reflections, or any combination of these. **Nonrigid transformations**, which generally distort the shape of a graph, include horizontal or vertical stretches and shrinks.

Vertical and Horizontal Translations

A **vertical translation** of the graph of $y = f(x)$ is a shift of the graph up or down in the coordinate plane. A **horizontal translation** is a shift of the graph to the left or the right. The following exploration will give you a good feel for what translations are and how they occur.

EXPLORATION 1 Introducing Translations

Set your viewing window to $[-5, 5]$ by $[-5, 15]$ and your graphing mode to sequential as opposed to simultaneous.

1. Graph the functions

$$y_1 = x^2 \qquad y_4 = y_1(x) - 2 = x^2 - 2$$
$$y_2 = y_1(x) + 3 = x^2 + 3 \qquad y_5 = y_1(x) - 4 = x^2 - 4$$
$$y_3 = y_1(x) + 1 = x^2 + 1$$

on the same screen. What effect do the $+3$, $+1$, -2, and -4 seem to have?

2. Graph the functions

$$y_1 = x^2 \qquad y_4 = y_1(x - 2) = (x - 2)^2$$
$$y_2 = y_1(x + 3) = (x + 3)^2 \qquad y_5 = y_1(x - 4) = (x - 4)^2$$
$$y_3 = y_1(x + 1) = (x + 1)^2$$

on the same screen. What effect do the $+3$, $+1$, -2, and -4 seem to have?

3. Repeat steps 1 and 2 for the functions $y_1 = x^3$, $y_1 = |x|$, and $y_1 = \sqrt{x}$. Do your observations agree with those you made after steps 1 and 2?

Technology Alert

In Exploration 1, the notation $y_1(x + 3)$ means the function y_1, evaluated at $x + 3$. It does not mean multiplication.

In general, *replacing* x by $x - c$ shifts the graph horizontally c units. Similarly, *replacing* y by $y - c$ shifts the graph vertically c units. If c is positive the shift is to the right or up; if c is negative the shift is to the left or down.

This is a nice, consistent rule that unfortunately gets complicated by the fact that the c for a vertical shift rarely shows up being subtracted from y. Instead, it usually shows up on the other side of the equal sign being *added* to $f(x)$. That leads us to the following rule, which only *appears* to be different for horizontal and vertical shifts:

Translations

Let c be a positive real number. Then the following transformations result in translations of the graph of $y = f(x)$:

Horizontal Translations

$y = f(x - c)$ a translation to the right by c units

$y = f(x + c)$ a translation to the left by c units

Vertical Translations

$y = f(x) + c$ a translation up by c units

$y = f(x) - c$ a translation down by c units

EXAMPLE 1 Vertical Translations

Describe how the graph of $y = |x|$ can be transformed to the graph of the given equation.

(a) $y = |x| - 4$ **(b)** $y = |x + 2|$

SOLUTION

(a) The equation is in the form $y = f(x) - 4$, a translation down by 4 units. See Figure 1.72.

(b) The equation is in the form $y = f(x + 2)$, a translation left by 2 units. See Figure 1.73.

Now try Exercise 3.

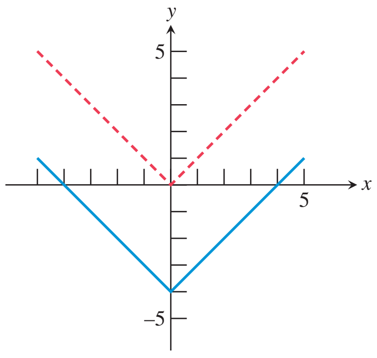


FIGURE 1.72 $y = |x| - 4$. (Example 1)

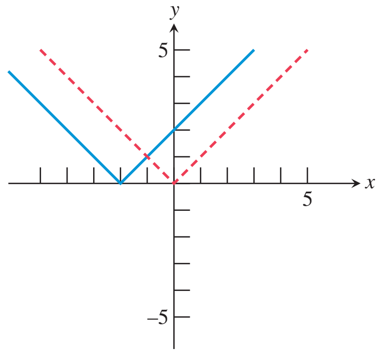


FIGURE 1.73 $y = |x + 2|$. (Example 1)

EXAMPLE 2 Finding Equations for Translations

Each view in Figure 1.74 shows the graph of $y_1 = x^3$ and a vertical or horizontal translation y_2. Write an equation for y_2 as shown in each graph.

SOLUTION

(a) $y_2 = x^3 - 3 = y_1(x) - 3$ (a vertical translation down by 3 units)

(b) $y_2 = (x + 2)^3 = y_1(x + 2)$ (a horizontal translation left by 2 units)

(c) $y_2 = (x - 3)^3 = y_1(x - 3)$ (a horizontal translation right by 3 units)

Now try Exercise 25.

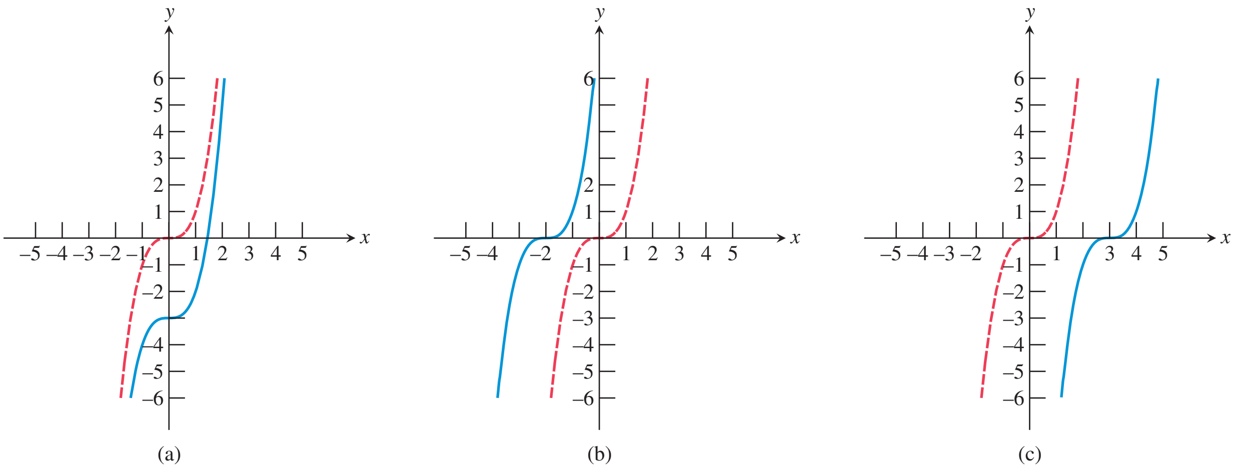


FIGURE 1.74 Translations of $y_1 = x^3$. (Example 2)

Reflections Across Axes

Points (x, y) and $(x, -y)$ are **reflections of each other across the x-axis**. Points (x, y) and $(-x, y)$ are **reflections of each other across the y-axis**. (See Figure 1.75.) Two points (or graphs) that are symmetric with respect to a line are **reflections of each other across that line**.

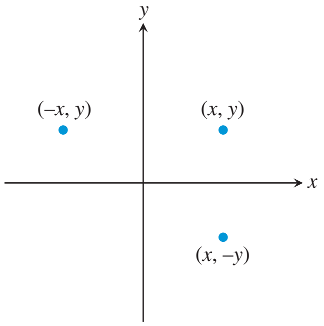


FIGURE 1.75 The point (x, y) and its reflections across the x-and y-axes.

Figure 1.75 suggests that a reflection across the x-axis results when y is replaced by $-y$, and a reflection across the y-axis results when x is replaced by $-x$.

Double Reflection

Note that a reflection through the origin is the result of reflections in both axes, performed in either order.

Reflections

The following transformations result in reflections of the graph of $y = f(x)$:

Across the x-axis

$$y = -f(x)$$

Across the y-axis

$$y = f(-x)$$

Through the origin

$$y = -f(-x)$$

EXAMPLE 3 Finding Equations for Reflections

Find an equation for the reflection of $f(x) = \dfrac{5x - 9}{x^2 + 3}$ across each axis.

(continued)

SOLUTION

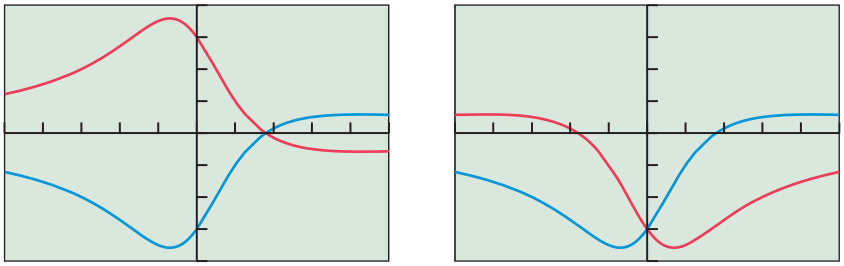
Solve Algebraically

Across the x-axis: $y = -f(x) = -\dfrac{5x - 9}{x^2 + 3} = \dfrac{9 - 5x}{x^2 + 3}$

Across the y-axis: $y = f(-x) = \dfrac{5(-x) - 9}{(-x)^2 + 3} = \dfrac{-5x - 9}{x^2 + 3}$

Support Graphically

The graphs in Figure 1.76 support our algebraic work.



[–5, 5] by [–4, 4]
(a)

[–5, 5] by [–4, 4]
(b)

FIGURE 1.76 Reflections of $f(x) = (5x - 9)/(x^2 + 3)$ across (a) the x-axis and (b) the y-axis. (Example 3)

Now try Exercise 29.

You might expect that odd and even functions, whose graphs already possess special symmetries, would exhibit special behavior when reflected across the axes. They do, as shown by Example 4 and Exercises 33 and 34.

EXAMPLE 4 Reflecting Even Functions

Prove that the graph of an even function remains unchanged when it is reflected across the y-axis.

SOLUTION Note that we can get plenty of graphical support for these statements by reflecting the graphs of various even functions, but what is called for here is **proof**, which will require algebra.

Let f be an even function; that is, $f(-x) = f(x)$ for all x in the domain of f. To reflect the graph of $y = f(x)$ across the y-axis, we make the transformation $y = f(-x)$. But $f(-x) = f(x)$ for all x in the domain of f, so this transformation results in $y = f(x)$. The graph of f therefore remains unchanged.

Now try Exercise 33.

Graphing Absolute Value Compositions

Given the graph of $y = f(x)$,

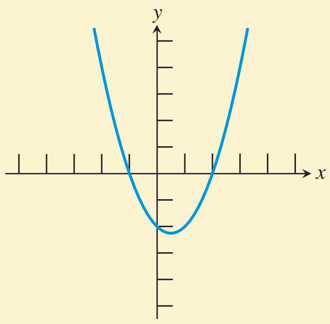
the graph of $y = |f(x)|$ can be obtained by reflecting the portion of the graph below the x-axis across the x-axis, leaving the portion above the x-axis unchanged;

the graph of $y = f(|x|)$ can be obtained by *replacing* the portion of the graph to the left of the y-axis by a reflection of the portion to the right of the y-axis across the y-axis, leaving the portion to the right of the y-axis unchanged. (The result will show even symmetry.)

Function compositions with absolute value can be realized graphically by reflecting portions of graphs, as you will see in the following Exploration.

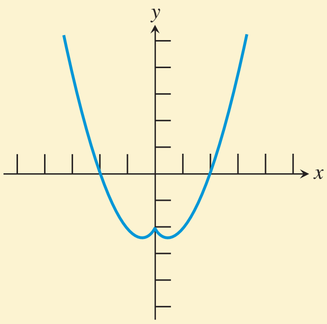
EXPLORATION 2 Compositions with Absolute Value

The graph of $y = f(x)$ is shown at the right. Match each of the graphs below with one of the following equations and use the language of function reflection to defend your match. Note that two of the graphs will not be used.

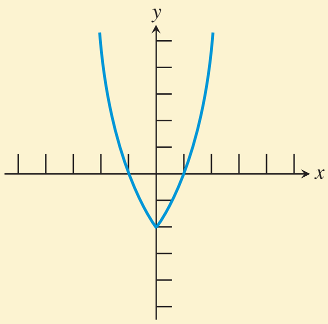


1. $y = |f(x)|$
2. $y = f(|x|)$
3. $y = -|f(x)|$
4. $y = |f(|x|)|$

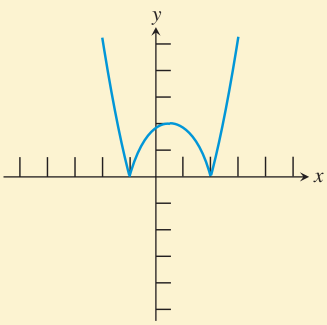
(A)



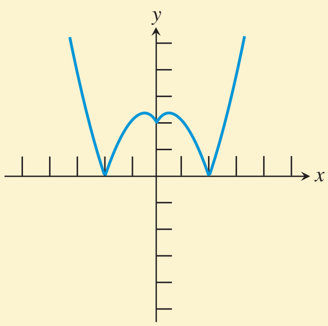
(B)



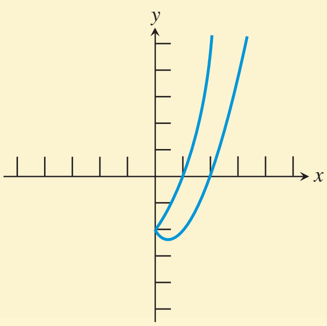
(C)



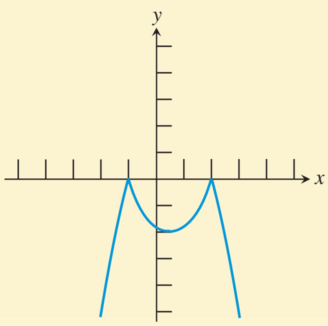
(D)



(E)



(F)



Vertical and Horizontal Stretches and Shrinks

We now investigate what happens when we multiply all the y-coordinates (or all the x-coordinates) of a graph by a fixed real number.

EXPLORATION 3 Introducing Stretches and Shrinks

Set your viewing window to $[-4.7, 4.7]$ by $[-1.1, 5.1]$ and your graphing mode to sequential as opposed to simultaneous.

1. Graph the functions

$$\begin{aligned} y_1 &= \sqrt{4 - x^2} \\ y_2 &= 1.5y_1(x) = 1.5\sqrt{4 - x^2} \\ y_3 &= 2y_1(x) = 2\sqrt{4 - x^2} \\ y_4 &= 0.5y_1(x) = 0.5\sqrt{4 - x^2} \\ y_5 &= 0.25y_1(x) = 0.25\sqrt{4 - x^2} \end{aligned}$$

on the same screen. What effect do the 1.5, 2, 0.5, and 0.25 seem to have?

2. Graph the functions

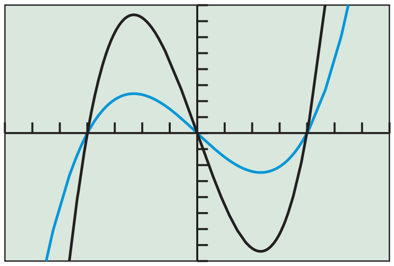
$$\begin{aligned} y_1 &= \sqrt{4 - x^2} \\ y_2 &= y_1(1.5x) = \sqrt{4 - (1.5x)^2} \\ y_3 &= y_1(2x) = \sqrt{4 - (2x)^2} \\ y_4 &= y_1(0.5x) = \sqrt{4 - (0.5x)^2} \\ y_5 &= y_1(0.25x) = \sqrt{4 - (0.25x)^2} \end{aligned}$$

on the same screen. What effect do the 1.5, 2, 0.5, and 0.25 seem to have?

Exploration 3 suggests that multiplication of x or y by a constant results in a horizontal or vertical stretching or shrinking of the graph.

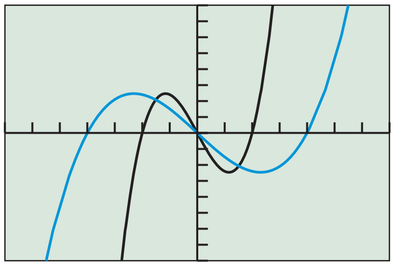
In general, *replacing x by x/c* distorts the graph horizontally by a factor of c. Similarly, *replacing y by y/c* distorts the graph vertically by a factor of c. If c is greater than 1 the distortion is a stretch; if c is less than 1 the distortion is a shrink.

As with translations, this is a nice, consistent rule that unfortunately gets complicated by the fact that the c for a vertical stretch or shrink rarely shows up as a divisor of y. Instead, it usually shows up on the other side of the equal sign as a *factor* multiplied by $f(x)$. That leads us to the following rule:



[–7, 7] by [–80, 80]

(a)



[–7, 7] by [–80, 80]

(b)

FIGURE 1.77 The graph of $y_1 = f(x) = x^3 - 16x$, shown with (a) a vertical stretch and (b) a horizontal shrink. (Example 5)

Stretches and Shrinks

Let c be a positive real number. Then the following transformations result in stretches or shrinks of the graph of $y = f(x)$:

Horizontal Stretches or Shrinks

$$y = f\left(\frac{x}{c}\right) \quad \begin{cases} \text{a stretch by a factor of } c & \text{if } c > 1 \\ \text{a shrink by a factor of } c & \text{if } c < 1 \end{cases}$$

Vertical Stretches or Shrinks

$$y = c \cdot f(x) \quad \begin{cases} \text{a stretch by a factor of } c & \text{if } c > 1 \\ \text{a shrink by a factor of } c & \text{if } c < 1 \end{cases}$$

EXAMPLE 5 Finding Equations for Stretches and Shrinks

Let C_1 be the curve defined by $y_1 = f(x) = x^3 - 16x$. Find equations for the following nonrigid transformations of C_1:

(a) C_2 is a vertical stretch of C_1 by a factor of 3.

(b) C_3 is a horizontal shrink of C_1 by a factor of 1/2.

SOLUTION

Solve Algebraically

(a) Denote the equation for C_2 by y_2. Then

$$\begin{aligned} y_2 &= 3 \cdot f(x) \\ &= 3(x^3 - 16x) \\ &= 3x^3 - 48x \end{aligned}$$

(b) Denote the equation for C_3 by y_3. Then

$$\begin{aligned} y_3 &= f\left(\frac{x}{1/2}\right) \\ &= f(2x) \\ &= (2x)^3 - 16(2x) \\ &= 8x^3 - 32x \end{aligned}$$

Support Graphically

The graphs in Figure 1.77 support our algebraic work. *Now try Exercise 39.*

Combining Transformations

Transformations may be performed in succession—one after another. If the transformations include stretches, shrinks, or reflections, the order in which the transformations are performed may make a difference. In those cases, be sure to pay particular attention to order.

EXAMPLE 6 Combining Transformations in Order

(a) The graph of $y = x^2$ undergoes the following transformations, in order. Find the equation of the graph that results.
- a horizontal shift 2 units to the right
- a vertical stretch by a factor of 3
- a vertical translation 5 units up

(b) Apply the transformations in (a) in the opposite order and find the equation of the graph that results.

SOLUTION

(a) Applying the transformations in order, we have

$$x^2 \Rightarrow (x - 2)^2 \Rightarrow 3(x - 2)^2 \Rightarrow 3(x - 2)^2 + 5.$$

Expanding the final expression, we get the function $y = 3x^2 - 12x + 17$.

(b) Applying the transformations in the opposite order, we have

$$x^2 \Rightarrow x^2 + 5 \Rightarrow 3(x^2 + 5) \Rightarrow 3((x - 2)^2 + 5).$$

Expanding the final expression, we get the function $y = 3x^2 - 12x + 27$.

The second graph is ten units higher than the first graph because the vertical stretch lengthens the vertical translation when the translation occurs first. Order often matters when stretches, shrinks, or reflections are involved.

Now try Exercise 47.

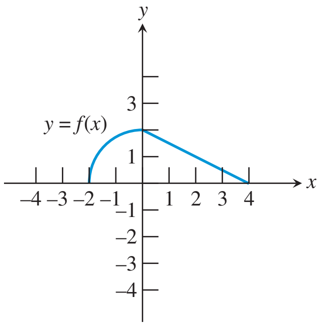


FIGURE 1.78 The graph of the function $y = f(x)$ in Example 7.

EXAMPLE 7 Transforming a Graph Geometrically

The graph of $y = f(x)$ is shown in Figure 1.78. Determine the graph of the composite function $y = 2f(x + 1) - 3$ by showing the effect of a sequence of transformations on the graph of $y = f(x)$.

SOLUTION

The graph of $y = 2f(x + 1) - 3$ can be obtained from the graph of $y = f(x)$ by the following sequence of transformations:

(a) a vertical stretch by a factor of 2 to get $y = 2f(x)$ (Figure 1.79a)

(b) a horizontal translation 1 unit to the left to get $y = 2f(x + 1)$ (Figure 1.79b)

(c) a vertical translation 3 units down to get $y = 2f(x + 1) - 3$ (Figure 1.79c)

(The order of the first two transformations can be reversed without changing the final graph.)

Now try Exercise 51.

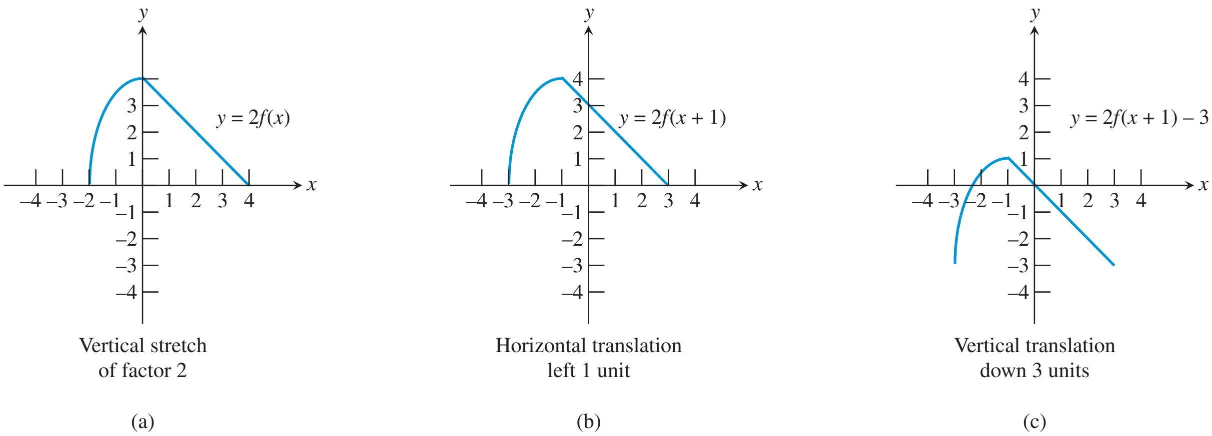


FIGURE 1.79 Transforming the graph of $y = f(x)$ in Figure 1.78 to get the graph of $y = 2f(x + 1) - 3$. (Example 7)

QUICK REVIEW 1.6 *(For help, go to Section A.2.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–6, write the expression as a binomial squared.

1. $x^2 + 2x + 1$

2. $x^2 - 6x + 9$

3. $x^2 + 12x + 36$

4. $4x^2 + 4x + 1$

5. $x^2 - 5x + \frac{25}{4}$

6. $4x^2 - 20x + 25$

In Exercises 7–10, perform the indicated operations and simplify.

7. $(x - 2)^2 + 3(x - 2) + 4$

8. $2(x + 3)^2 - 5(x + 3) - 2$

9. $(x - 1)^3 + 3(x - 1)^2 - 3(x - 1)$

10. $2(x + 1)^3 - 6(x + 1)^2 + 6(x + 1) - 2$

SECTION 1.6 EXERCISES

In Exercises 1–8, describe how the graph of $y = x^2$ can be transformed to the graph of the given equation.

1. $y = x^2 - 3$

2. $y = x^2 + 5.2$

3. $y = (x + 4)^2$

4. $y = (x - 3)^2$

5. $y = (100 - x)^2$

6. $y = x^2 - 100$

7. $y = (x - 1)^2 + 3$

8. $y = (x + 50)^2 - 279$

In Exercises 9–12, describe how the graph of $y = \sqrt{x}$ can be transformed to the graph of the given equation.

9. $y = -\sqrt{x}$

10. $y = \sqrt{x - 5}$

11. $y = \sqrt{-x}$

12. $y = \sqrt{3 - x}$

In Exercises 13–16, describe how the graph of $y = x^3$ can be transformed to the graph of the given equation.

13. $y = 2x^3$

14. $y = (2x)^3$

15. $y = (0.2x)^3$

16. $y = 0.3x^3$

In Exercises 17–20, describe how to transform the graph of f into the graph of g.

17. $f(x) = \sqrt{x+2}$ and $g(x) = \sqrt{x-4}$

18. $f(x) = (x-1)^2$ and $g(x) = -(x+3)^2$

19. $f(x) = (x-2)^3$ and $g(x) = -(x+2)^3$

20. $f(x) = |2x|$ and $g(x) = 4|x|$

In Exercises 21–24, sketch the graphs of f, g, and h by hand. Support your answers with a grapher.

21. $f(x) = (x+2)^2$
$g(x) = 3x^2 - 2$
$h(x) = -2(x-3)^2$

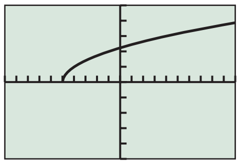
22. $f(x) = x^3 - 2$
$g(x) = (x+4)^3 - 1$
$h(x) = 2(x-1)^3$

23. $f(x) = \sqrt[3]{x+1}$
$g(x) = 2\sqrt[3]{x} - 2$
$h(x) = -\sqrt[3]{x-3}$

24. $f(x) = -2|x| - 3$
$g(x) = 3|x+5| + 4$
$h(x) = |3x|$

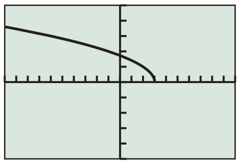
In Exercises 25–28, the graph is that of a function $y = f(x)$ that can be obtained by transforming the graph of $y = \sqrt{x}$. Write a formula for the function f.

25.



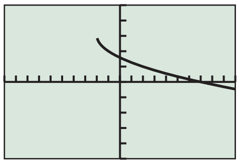
[–10, 10] by [–5, 5]

26.



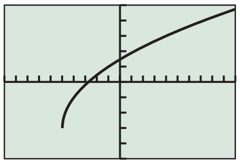
[–10, 10] by [–5, 5]

27.



[–10, 10] by [–5, 5]

28.



[–10, 10] by [–5, 5]
Vertical stretch = 2

In Exercises 29–32, find the equation of the reflection of f across **(a)** the x-axis and **(b)** the y-axis.

29. $f(x) = x^3 - 5x^2 - 3x + 2$

30. $f(x) = 2\sqrt{x+3} - 4$

31. $f(x) = \sqrt[3]{8x}$

32. $f(x) = 3|x+5|$

33. Reflecting Odd Functions Prove that the graph of an odd function is the same when reflected across the x-axis as it is when reflected across the y-axis.

34. Reflecting Odd Functions Prove that if an odd function is reflected about the y-axis and then reflected again about the x-axis, the result is the original function.

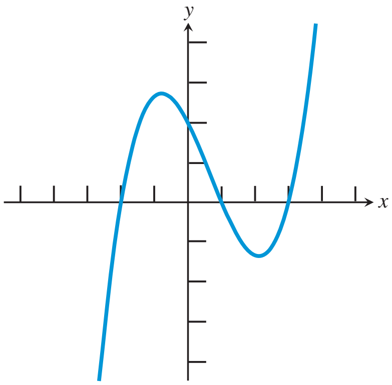
Exercises 35–38 refer to the graph of $y = f(x)$ shown at the top of the next column. In each case, sketch a graph of the new function.

35. $y = |f(x)|$

36. $y = f(|x|)$

37. $y = -f(|x|)$

38. $y = |f(|x|)|$



In Exercises 39–42, transform the given function by **(a)** a vertical stretch by a factor of 2, and **(b)** a horizontal shrink by a factor of 1/3.

39. $f(x) = x^3 - 4x$

40. $f(x) = |x+2|$

41. $f(x) = x^2 + x - 2$

42. $f(x) = \dfrac{1}{x+2}$

In Exercises 43–46, describe a basic graph and a sequence of transformations that can be used to produce a graph of the given function.

43. $y = 2(x-3)^2 - 4$

44. $y = -3\sqrt{x+1}$

45. $y = (3x)^2 - 4$

46. $y = -2|x+4| + 1$

In Exercises 47–50, a graph G is obtained from a graph of y by the sequence of transformations indicated. Write an equation whose graph is G.

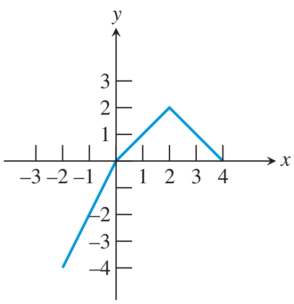
47. $y = x^2$: a vertical stretch by a factor of 3, then a shift right 4 units.

48. $y = x^2$: a shift right 4 units, then a vertical stretch by a factor of 3.

49. $y = |x|$: a shift left 2 units, then a vertical stretch by a factor of 2, and finally a shift down 4 units.

50. $y = |x|$: a shift left 2 units, then a horizontal shrink by a factor of 1/2, and finally a shift down 4 units.

Exercises 51–54 refer to the function f whose graph is shown below.



51. Sketch the graph of $y = 2 + 3f(x+1)$.

52. Sketch the graph of $y = -f(x+1) + 1$.

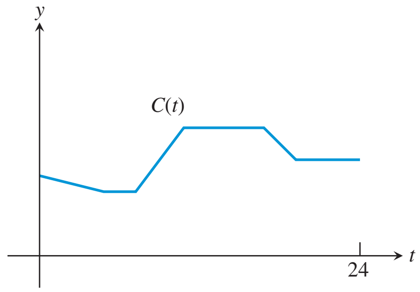
53. Sketch the graph of $y = f(2x)$.

54. Sketch the graph of $y = 2f(x-1) + 2$.

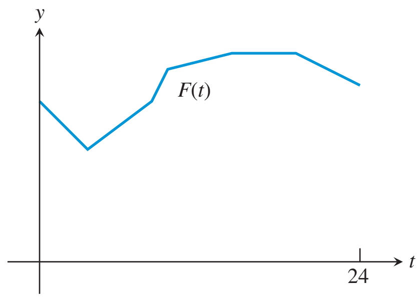
55. Writing to Learn Graph some examples to convince yourself that a reflection and a translation can have a different effect when combined in one order than when combined in the opposite order. Then explain in your own words why this can happen.

56. Writing to Learn Graph some examples to convince yourself that vertical stretches and shrinks do not affect a graph's x-intercepts. Then explain in your own words why this is so.

57. Celsius vs. Fahrenheit The graph shows the temperature in degrees Celsius in Windsor, Ontario, for one 24-hour period. Describe the transformations that convert this graph to one showing degrees Fahrenheit. [*Hint*: $F(t) = (9/5)C(t) + 32$.]



58. Fahrenheit vs. Celsius The graph shows the temperature in degrees Fahrenheit in Mt. Clemens, Michigan, for one 24-hour period. Describe the transformations that convert this graph to one showing degrees Celsius.
[*Hint*: $F(t) = (9/5)C(t) + 32$.]



Standardized Test Questions

59. True or False The function $y = f(x + 3)$ represents a translation to the right by 3 units of the graph of $y = f(x)$. Justify your answer.

60. True or False The function $y = f(x) - 4$ represents a translation down 4 units of the graph of $y = f(x)$. Justify your answer.

In Exercises 61–64, you may use a graphing calculator to answer the question.

61. Multiple Choice Given a function f, which of the following represents a vertical stretch by a factor of 3?

(A) $y = f(3x)$ **(B)** $y = f(x/3)$
(C) $y = 3f(x)$ **(D)** $y = f(x)/3$
(E) $y = f(x) + 3$

62. Multiple Choice Given a function f, which of the following represents a horizontal translation of 4 units to the right?

(A) $y = f(x) + 4$ **(B)** $y = f(x) - 4$
(C) $y = f(x + 4)$ **(D)** $y = f(x - 4)$
(E) $y = 4f(x)$

63. Multiple Choice Given a function f, which of the following represents a vertical translation of 2 units upward, followed by a reflection across the y-axis?

(A) $y = f(-x) + 2$ **(B)** $y = 2 - f(x)$
(C) $y = f(2 - x)$ **(D)** $y = -f(x - 2)$
(E) $y = f(x) - 2$

64. Multiple Choice Given a function f, which of the following represents reflection across the x-axis, followed by a horizontal shrink by a factor of 1/2?

(A) $y = -2f(x)$ **(B)** $y = -f(x)/2$
(C) $y = f(-2x)$ **(D)** $y = -f(x/2)$
(E) $y = -f(2x)$

Explorations

65. International Finance Table 1.11 shows the (adjusted closing) price of a share of stock in Dell Computer for each month of 2008.

Table 1.11 Dell Computer

Month	Price ($)
1	20.04
2	19.90
3	19.92
4	18.63
5	23.06
6	21.88
7	24.57
8	21.73
9	16.48
10	12.20
11	11.17
12	10.24

Source: Yahoo! Finance.

(a) Graph price (y) as a function of month (x) as a line graph, connecting the points to make a continuous graph.

(b) Explain what transformation you would apply to this graph to produce a graph showing the price of the stock in Japanese yen.

66. Group Activity Get with a friend and graph the function $y = x^2$ on both your graphers. Apply a horizontal or vertical stretch or shrink to the function on one of the graphers. Then change the *window* of that grapher to make the two graphs look the same. Can you formulate a general rule for how to find the window?

Extending the Ideas

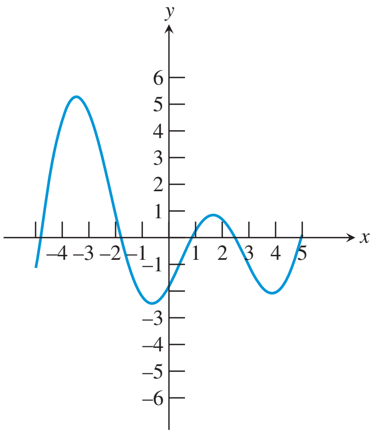
67. The Absolute Value Transformation Graph the function $f(x) = x^4 - 5x^3 + 4x^2 + 3x + 2$ in the viewing window $[-5, 5]$ by $[-10, 10]$. (Put the equation in Y1.)

(a) Study the graph and try to predict what the graph of $y = |f(x)|$ will look like. Then turn Y1 off and graph Y2 = abs (Y1). Did you predict correctly?

(b) Study the original graph again and try to predict what the graph of $y = f(|x|)$ will look like. Then turn Y1 off and graph Y2 = Y1(abs (X)). Did you predict correctly?

(c) Given the graph of $y = g(x)$ shown below, sketch a graph of $y = |g(x)|$.

(d) Given the graph of $y = g(x)$ shown below, sketch a graph of $y = g(|x|)$.



68. Parametric Circles and Ellipses Set your grapher to parametric and radian mode and your window as follows:

Tmin = 0, Tmax = 7, Tstep = 0.1

Xmin = −4.7, Xmax = 4.7, Xscl = 1

Ymin = −3.1, Ymax = 3.1, Yscl = 1

(a) Graph the parametric equations $x = \cos t$ and $y = \sin t$. You should get a circle of radius 1.

(b) Use a transformation of the parametric function of x to produce the graph of an ellipse that is 4 units wide and 2 units tall.

(c) Use a transformation of both parametric functions to produce a circle of radius 3.

(d) Use a transformation of both functions to produce an ellipse that is 8 units wide and 4 units tall.

(You will learn more about ellipses in Chapter 8.)