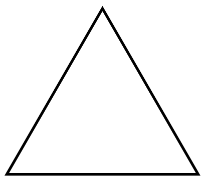
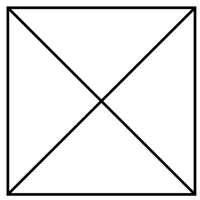
■ Section 1.7 Modeling with Functions

Exploration 1

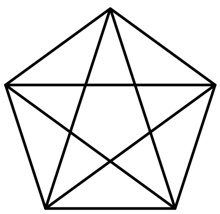
1.



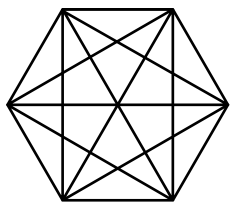
$n = 3;\ d = 0$



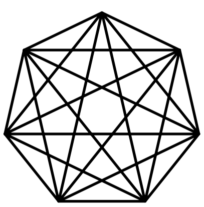
$n = 4;\ d = 2$



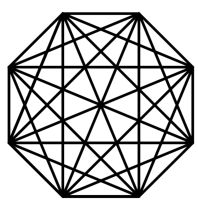
$n = 5;\ d = 5$



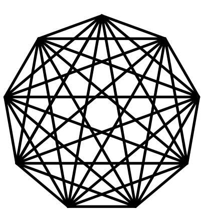
$n = 6;\ d = 9$



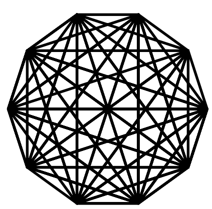
$n = 7;\ d = 14$



$n = 8;\ d = 20$

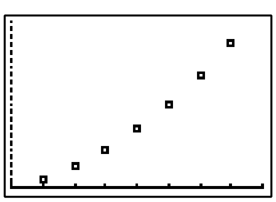


$n = 9;\ d = 27$



$n = 10;\ d = 35$

2.



[3, 11] by [0, 40]

3. Linear: $r^2 = 0.9758$
Power: $r^2 = 0.9903$
Quadratic: $R^2 = 1$
Cubic: $R^2 = 1$
Quartic: $R^2 = 1$

4. The best-fit curve is quadratic: $y = 0.5x^2 - 1.5x$. The cubic and quartic regressions give this same curve.

5. Since the quadratic curve fits the points perfectly, there is nothing to be gained by adding a cubic term or a quartic term. The coefficients of these terms in the regressions are zero.

6. $y = 0.5x^2 - 1.5x$. At $x = 128$,
$y = 0.5(128)^2 - 1.5(128) = 8000$

Quick Review 1.7

1. $h = 2(A/b)$

2. $h = 2A/(b_1 + b_2)$

3. $h = V/(\pi r^2)$

4. $h = 3V/(\pi r^2)$

5. $r = \sqrt[3]{\dfrac{3V}{4\pi}}$

6. $r = \sqrt{\dfrac{A}{4\pi}}$

7. $h = \dfrac{A - 2\pi r^2}{2\pi r} = \dfrac{A}{2\pi r} - r$

8. $t = I/(Pr)$

9. $P = \dfrac{A}{(1 + r/n)^{nt}} = A\left(1 + \dfrac{r}{n}\right)^{-nt}$

10. $t = \sqrt{\dfrac{2(H - s)}{g}}$

Section 1.7 Exercises

1. $3x + 5$

2. $3(x + 5)$

3. $0.17x$

4. $0.05x + 4$

5. $A = \ell w = (x + 12)(x)$

6. $A = \dfrac{1}{2}bh = \dfrac{1}{2}(x)(x + 2)$

7. $x + 0.045x = (1 + 0.045)x = 1.045x$

8. $x - 0.03x = (1 - 0.03)x = 0.97x$

9. $x - 0.40x = 0.60x$

10. $x + 0.0875x = 1.0875x$

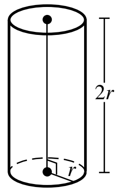
11. Let C be the total cost and n be the number of items produced; $C = 34{,}500 + 5.75n$.

12. Let C be the total cost and n be the number of items produced; $C = (1.09)28{,}000 + 19.85n$.

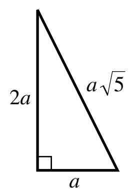
13. Let R be the revenue and n be the number of items sold; $R = 3.75n$.

14. Let P be the profit, and s be the amount of sales; then $P = 200{,}000 + 0.12s$.

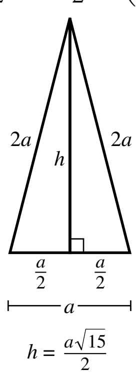
15. The basic formula for the volume of a right circular cylinder is $V = \pi r^2 h$, where r is the radius and h is height. Since height equals diameter $(h = d)$ and the diameter is two times r $(d = 2r)$, we know $h = 2r$. Then, $V = \pi r^2(2r) = 2\pi r^3$.



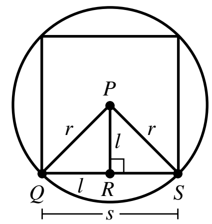
16. Let c = hypotenuse, a = "short" side, and b = "long" side. Then $c^2 = a^2 + b^2 = a^2 + (2a)^2 = a^2 + 4a^2 = 5a^2$, so $c = \sqrt{5a^2} = a\sqrt{5}$.



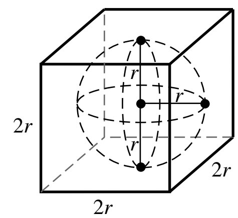
17. Let a be the length of the base. Then the other two sides of the triangle have length two times the base, or $2a$. Since the triangle is isoceles, a perpendicular dropped from the "top" vertex to the base is perpendicular. As a result, $h^2 + \left(\frac{a}{2}\right)^2 = (2a)^2$, or $h^2 = 4a^2 - \frac{a^2}{4} = \frac{16a^2 - a^2}{4}$ $= \frac{15a^2}{4}$, so $h = \sqrt{\frac{15a^2}{4}} = \frac{a\sqrt{15}}{2}$. The triangle's area is $A = \frac{1}{2}bh = \frac{1}{2}(a)\left(\frac{a\sqrt{15}}{2}\right) = \frac{a^2\sqrt{15}}{4}$.



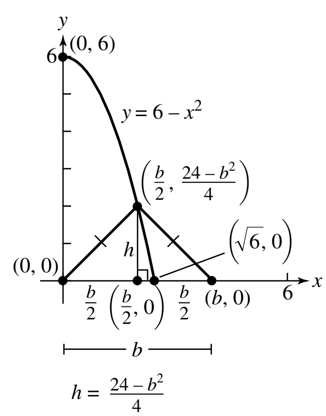
18. Since P lies at the center of the square and the circle, we know that segment $\overline{PR} = \overline{QR} = \overline{RS}$. Let ℓ be the length of these segments. Then, $\ell^2 + \ell^2 = r^2$, $2\ell^2 = r^2$, $\ell^2 = \frac{r^2}{2}$, $\ell = \sqrt{\frac{r^2}{2}} = \frac{r}{\sqrt{2}} = \frac{r\sqrt{2}}{2}$.
Since each side of the square is two times ℓ, we know that $s = 2\ell = \left(\frac{r\sqrt{2}}{2}\right)2 = r\sqrt{2}$.
As a result, $A = s^2 = (r\sqrt{2})^2 = r^2 \cdot 2 = 2r^2$.



19. Let r be the radius of the sphere. Since the sphere is tangent to all six faces of the cube, we know that the height (and width, and depth) of the cube is equal to the sphere's diameter, which is two times r $(2r)$. The surface area of the cube is the sum of the area of all six faces, which equals $2r \cdot 2r = 4r^2$. Thus, $A = 6 \cdot 4r^2 = 24r^2$.



20. From our graph, we see that y provides the height of our triangle, i.e., $h = y$ when $x = \frac{b}{2}$. Since $y = 6 - x^2$ $= 6 - \left(\frac{b}{2}\right)^2 = 6 - \frac{b^2}{4} = \frac{24 - b^2}{4}$, $h = \frac{24 - b^2}{4}$.
The area of the triangle is $A = \frac{1}{2}bh = \frac{1}{2}b\left(\frac{24 - b^2}{4}\right)$ $= \frac{24b - b^3}{8}$.



21. Solving $x + 4x = 620$ gives $x = 124$, so $4x = 496$. The two numbers are 124 and 496.

22. $x + 2x + 3x = 714$, so $x = 119$; the second and third numbers are 238 and 357.

23. $1.035x = 36{,}432$, so $x = 35{,}200$

24. $1.023x = 184.0$, so $x = 179.9$.

25. $182 = 52t$, so $t = 3.5$ hr.

26. $560 = 45t + 55(t + 2)$, so $t = 4.5$ hours on local highways.

27. $0.60(33) = 19.8$; $0.75(27) = 20.25$. The \$33 shirt sells for \$19.80. The \$27 shirt sells for \$20.25. The \$33 shirt is a better bargain, because the sale price is cheaper.

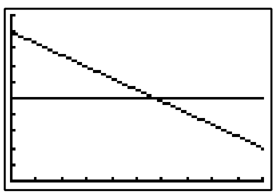
28. Let x be gross sales. For the second job to be more attractive than the first, we need
$20{,}000 + 0.07x > 25{,}000 + 0.05x$, $0.02x > 5000$,
$x > \frac{5000}{0.02} = \$250{,}000$.
Gross sales would have to exceed \$250,000.

29. $71\,065\,000(1 + x) = 82\,400\,000$
$$71\,065\,000x = 82\,400\,000 - 71\,065\,000$$
$$x = \frac{82\,400\,000 - 71\,065\,000}{71\,065\,000} \approx 0.1595$$
There was a 15.95% increase in sales.

30. $26\,650\,000(1 + x) = 30\,989\,000$
$$26\,650\,000x = 30\,989\,000 - 26\,650\,000$$
$$x = \frac{30\,989\,000 - 26\,650\,000}{26\,650\,000} \approx 0.1628$$
Shipments of personal computers grew 16.28%.

31. **(a)** $0.10x + 0.45(100 - x) = 0.25(100)$.

(b) Graph $y_1 = 0.1x + 0.45(100 - x)$ and $y_2 = 25$. Use $x \approx 57.14$ gallons of the 10% solution and about 42.86 gal of the 45% solution.



[0, 100] by [0, 50]

32. Solve $0.20x + 0.35(25 - x) = 0.26(25)$. Use $x = 15$ liters of the 20% solution and 10 liters of the 35% solution.

33. **(a)** The height of the box is x, and the base measures $10 - 2x$ by $18 - 2x$.
$V(x) = x(10 - 2x)(18 - 2x)$

(b) Because one side of the original piece of cardboard measures 10 in., $2x$ must be greater than 0 but less than 10, so that $0 < x < 5$. The domain of $V(x)$ is $(0, 5)$.

(c) Graphing $V(x)$ produces a cubic-function curve that between $x = 0$ and $x = 5$ has a maximum at approximately (2.06, 168.1). The cut-out squares should measure approximately 2.06 in. by 2.06 in.

34. Solve $2x + 2(x + 16) = 136$. Two pieces that are $x = 26$ ft long are needed, along with two 42 ft pieces.

35. Equation of the parabola, to pass through $(-16, 8)$ and $(16, 8)$:
$y = kx^2$
$8 = k(\pm 16)^2$
$k = \frac{8}{256} = \frac{1}{32}$
$y = \frac{1}{32}x^2$
y-coordinate of parabola 8 in. from center:
$y = \frac{1}{32}(8)^2 = 2$
From that point to the top of the dish is $8 - 2 = 6$ in.

36. Solve $2x + 2(x + 3) = 54$. This gives $x = 12$; the room is 12 ft $\times$ 15 ft.

37. Original volume of water:
$V_0 = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(9)^2(24) \approx 2035.75 \text{ in.}^3$
Volume lost through faucet:
$V_1 = \text{time} \times \text{rate} = (120 \text{ sec})(5 \text{ in.}^3/\text{sec}) = 600 \text{ in.}^3$
Find volume:
$V_f = V_0 - V_1 = 2035.75 - 600 = 1435.75$
Since the final cone-shaped volume of water has radius and height in a 9-to-24 ratio, or $r = \frac{3}{8}h$:
$$V_f = \frac{1}{3}\pi\left(\frac{3}{8}h\right)^2 h = \frac{3}{64}\pi h^3 = 1435.75$$
Solving, we obtain $h \approx 21.36$ in.

38. Solve $900 = 0.07x + 0.085(12{,}000 - x)$.
$x = 8000$ dollars was invested at 7%; the other \$4000 was invested at 8.5%.

39. Bicycle's speed in feet per second:
$(2 \times \pi \times 16 \text{ in./rot})(2 \text{ rot/sec}) = 64\pi \text{ in./sec}$
Unit conversion:
$(64\pi \text{ in./sec})\left(\frac{1}{12} \text{ ft/in.}\right)\left(\frac{1}{5280} \text{ mi/ft}\right)(3600 \text{ sec/hr})$
≈ 11.42 mi/hr

40. Solve $1571 = 0.055x + 0.083(25{,}000 - x)$.
$x = 18{,}000$ dollars was invested at 5.5%; the other \$7000 was invested at 8.3%.

41. True. The correlation coefficient is close to 1 (or -1) if there is a good fit. A correlation coefficient near 0 indicates a very poor fit.

42. False. The graph over time of the height of a freely falling object is a parabola. A quadratic regression is called for.

43. The pattern of points is S-shaped, which suggests a cubic model. The answer is C.

44. The points appear to lie along a straight line. The answer is A.

45. The points appear to lie along an upward-opening parabola. The answer is B.

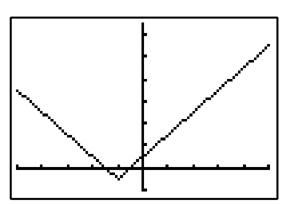
46. The pattern of points looks sinusoidal. The answer is E.

47. **(a)** $C = 100{,}000 + 30x$

(b) $R = 50x$

(c) $100{,}000 + 30x = 50x$
$$100{,}000 = 20x$$
$$x = 5000 \text{ pairs of shoes}$$

(d) Graph $y_1 = 100{,}000 + 30x$ and $y_2 = 50x$; these graphs cross when $x = 5000$ pairs of shoes. The point of intersection corresponds to the break-even point, where $C = R$.



$[-10, 10]$ by $[-2, 18]$

48. Solve $48{,}814.20 = x + 0.12x + 0.03x + 0.004x$. Then $48{,}814.20 = 1.154x$, so $x = 42{,}300$ dollars.

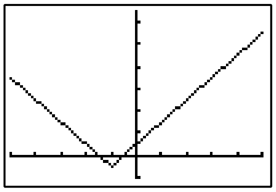
49. **(a)** $y_1 = u(x) = 125{,}000 + 23x$.

(b) $y_2 = s(x) = 125{,}000 + 23x + 8x = 125{,}000 + 31x$.

(c) $y_3 = r_u(x) = 56x$.

(d) $y_4 = R_s(x) = 79x$.

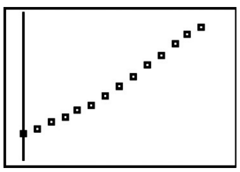
(e)



$[-10, 10]$ by $[-2, 18]$

(f) You should recommend stringing the rackets; fewer strung rackets need to be sold to begin making a profit (since the intersection of y_2 and y_4 occurs for smaller x than the intersection of y_1 and y_3).

50. **(a)**



[–1, 15] by [9, 16]

(b) $y = 0.409x + 9.861$

(c) $r = 0.993$, so the linear model is appropriate.

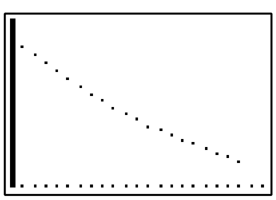
(d) $y = 0.012x^2 + 0.247x + 10.184$

(e) $r^2 = 0.998$, so a quadratic model is appropriate.

(f) The linear prediction is 18.04 and the quadratic prediction is 19.92. Despite the fact that both models look good for the data, the predictions differ by 1.88. One or both of them must be ineffective, as they both cannot be right.

(g) The linear regression and the quadratic regression are very close from $x = 0$ to $x = 13$. The quadratic regression begins to veer away from the linear regression at $x = 13$. Since there are no data points beyond $x = 13$, it is difficult to know which is accurate.

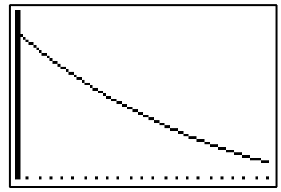
51. **(a)**



$[0, 22]$ by $[100, 200]$

(b) List L3 = {112.3, 106.5, 101.5, 96.6, 92.0, 87.2, 83.1, 79.8, 75.0, 71.7, 68, 64.1, 61.5, 58.5, 55.9, 53.0, 50.8, 47.9, 45.2, 43.2}

(c) The regression equation is $y = 118.07 \times 0.951^x$. It fits the data extremely well.



$[0, 22]$ by $[100, 200]$

52. Answers will vary in **(a)–(e)**, depending on the conditions of the experiment.

(f) Some possible answers: the thickness of the liquid, the darkness of the liquid, the type of cup it is in, the amount of surface exposed to the air, the specific heat of the substance (a technical term that may have been learned in physics), etc.