# 1.7 Modeling with Functions

### What you'll learn about

- Functions from Formulas
- Functions from Graphs
- Functions from Verbal Descriptions
- Functions from Data

### ... and why

Using a function to model a variable under observation in terms of another variable often allows one to make predictions in practical situations, such as predicting the future growth of a business based on known data.

## Functions from Formulas

Now that you have learned more about what functions are and how they behave, we want to return to the modeling theme of Section 1.1. In that section we stressed that one of the goals of this course was to become adept at using numerical, algebraic, and graphical models of the real world in order to solve problems. We now want to focus your attention more precisely on modeling with *functions*.

You have already seen quite a few formulas in the course of your education. Formulas involving two variable quantities always relate those variables implicitly, and quite often the formulas can be solved to give one variable explicitly as a function of the other. In this book we will use a variety of formulas to pose and solve problems algebraically, although we will not assume prior familiarity with those formulas that we borrow from other subject areas (like physics or economics). We *will* assume familiarity with certain key formulas from mathematics.

### EXAMPLE 1 Forming Functions from Formulas

Write the area $A$ of a circle as a function of its

**(a)** radius $r$.

**(b)** diameter $d$.

**(c)** circumference $C$.

**SOLUTION**

**(a)** The familiar area formula from geometry gives $A$ as a function of $r$:

$$A = \pi r^2$$

**(b)** This formula is not so familiar. However, we know that $r = d/2$, so we can substitute that expression for $r$ in the area formula:

$$A = \pi r^2 = \pi(d/2)^2 = (\pi/4)d^2$$

**(c)** Since $C = 2\pi r$, we can solve for $r$ to get $r = C/(2\pi)$. Then substitute to get $A$: $A = \pi r^2 = \pi(C/(2\pi))^2 = \pi C^2/(4\pi^2) = C^2/(4\pi)$. *Now try Exercise 19.*

### EXAMPLE 2 A Maximum Value Problem

A square of side $x$ inches is cut out of each corner of an 8 in. by 15 in. piece of cardboard and the sides are folded up to form an open-topped box (Figure 1.80).

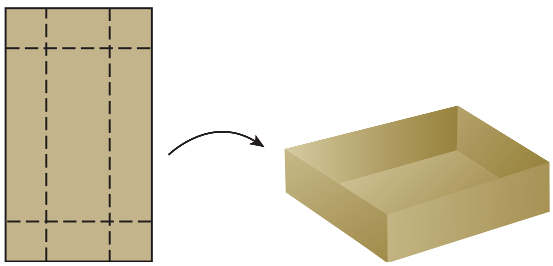
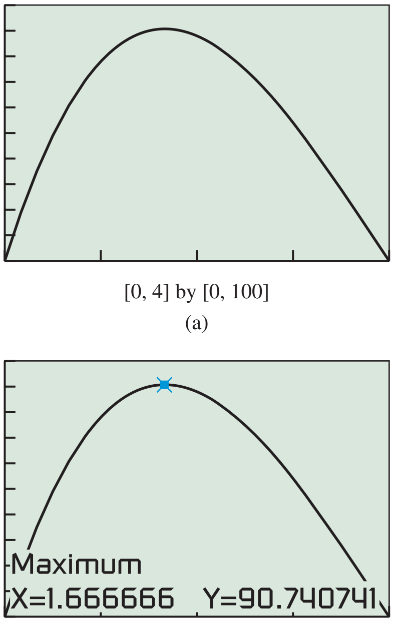


**FIGURE 1.80** An open-topped box made by cutting the corners from a piece of cardboard and folding up the sides. (Example 2)




[0, 4] by [0, 100]
(a)

[0, 4] by [0, 100]
(b)

**FIGURE 1.81** The graph of the volume of the box in Example 2.

**(a)** Write the volume $V$ of the box as a function of $x$.

**(b)** Find the domain of $V$ as a function of $x$. (Note that the model imposes restrictions on $x$.)

**(c)** Graph $V$ as a function of $x$ over the domain found in part (b) and use the maximum finder on your grapher to determine the maximum volume such a box can hold.

**(d)** How big should the cut-out squares be in order to produce the box of maximum volume?

**SOLUTION**

**(a)** The box will have a base with sides of width $8 - 2x$ and length $15 - 2x$. The depth of the box will be $x$ when the sides are folded up. Therefore $V = x(8 - 2x)(15 - 2x)$.

**(b)** The formula for $V$ is a polynomial with domain all reals. However, the depth $x$ must be nonnegative, as must the width of the base, $8 - 2x$. Together, these two restrictions yield a domain of [0, 4]. (The endpoints give a box with no volume, which is as mathematically feasible as other zero concepts.)

**(c)** The graph is shown in Figure 1.81. The maximum finder shows that the maximum occurs at the point $(5/3, 90.74)$. The maximum volume is about 90.74 in.$^3$.

**(d)** Each square should have sides of one-and-two-thirds inches.

***Now try Exercise 33.***

## Functions from Graphs

When "thinking graphically" becomes a genuine part of your problem-solving strategy, it is sometimes actually easier to start with the graphical model than it is to go straight to the algebraic formula. The graph provides valuable information about the function.

### EXAMPLE 3 Protecting an Antenna

A small satellite dish is packaged with a cardboard cylinder for protection. The parabolic dish is 24 in. in diameter and 6 in. deep, and the diameter of the cardboard cylinder is 12 in. How tall must the cylinder be to fit in the middle of the dish and be flush with the top of the dish? (See Figure 1.82.)

**SOLUTION**

#### Solve Algebraically

The diagram in Figure 1.82a showing the cross section of this 3-dimensional problem is also a 2-dimensional graph of a quadratic function. We can transform our basic function $y = x^2$ with a vertical shrink so that it goes through the points $(12, 6)$ and $(-12, 6)$, thereby producing a graph of the parabola in the coordinate plane (Figure 1.82b).

$$y = kx^2 \quad \text{Vertical shrink}$$
$$6 = k(\pm 12)^2 \quad \text{Substitute } x = \pm 12,\ y = 6.$$
$$k = \frac{6}{144} = \frac{1}{24} \quad \text{Solve for } k.$$

Thus, $y = \dfrac{1}{24}x^2$.

*(continued)*

To find the height of the cardboard cylinder, we first find the $y$-coordinate of the parabola 6 inches from the center, that is, when $x = 6$:

$$y = \frac{1}{24}(6)^2 = 1.5$$

From that point to the top of the dish is $6 - 1.5 = 4.5$ in. ***Now try Exercise 35.***

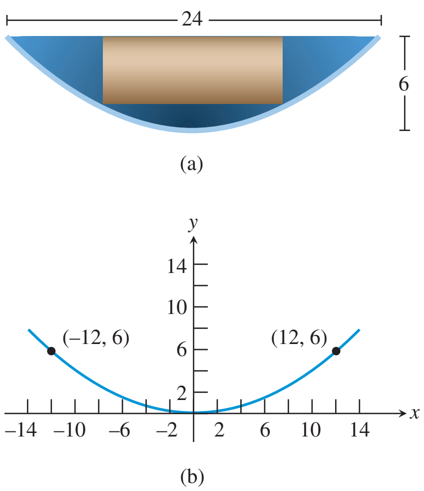



**FIGURE 1.82** (a) A parabolic satellite dish with a protective cardboard cylinder in the middle for packaging. (b) The parabola in the coordinate plane. (Example 3)

Although Example 3 serves nicely as a "functions from graphs" example, it is also an example of a function that must be constructed by gathering relevant information from a verbal description and putting it together in the right way. People who do mathematics for a living are accustomed to confronting that challenge regularly as a necessary first step in modeling the real world. In honor of its importance, we have saved it until last to close out this chapter in style.

## Functions from Verbal Descriptions

There is no fail-safe way to form a function from a verbal description. It can be hard work, frequently a good deal harder than the mathematics required to solve the problem once the function has been found. The 4-step problem-solving process in Section 1.1 gives you several valuable tips, perhaps the most important of which is to *read* the problem carefully. Understanding what the words say is critical if you hope to model the situation they describe.

### EXAMPLE 4 Finding the Model and Solving

Grain is leaking through a hole in a storage bin at a constant rate of 8 cubic inches per minute. The grain forms a cone-shaped pile on the ground below. As it grows, the height of the cone always remains equal to its radius. If the cone is one foot tall now, how tall will it be in one hour?

**SOLUTION** Reading the problem carefully, we realize that the formula for the volume of the cone is needed (Figure 1.83). From memory or by looking it up, we get the formula $V = (1/3)\pi r^2 h$. A careful reading also reveals that the height and the radius are always equal, so we can get volume directly as a function of height: $V = (1/3)\pi h^3$.

When $h = 12$ in., the volume is $V = (\pi/3)(12)^3 = 576\pi$ in.$^3$.

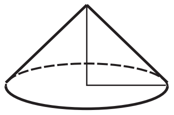


**FIGURE 1.83** A cone with equal height and radius. (Example 4)

One hour later, the volume will have grown by $(60 \text{ min})(8 \text{ in.}^3/\text{min}) = 480 \text{ in.}^3$. The total volume of the pile at that point will be $(576\pi + 480) \text{ in.}^3$. Finally, we use the volume formula once again to solve for $h$:

$$\frac{1}{3}\pi h^3 = 576\pi + 480$$

$$h^3 = \frac{3(576\pi + 480)}{\pi}$$

$$h = \sqrt[3]{\frac{3(576\pi + 480)}{\pi}}$$

$$h \approx 12.98 \text{ inches}$$

*Now try Exercise 37.*

## EXAMPLE 5 Letting Units Work for You

How many rotations does a 15-in. (radius) tire make per second on a sport utility vehicle traveling 70 mph?

**SOLUTION** It is the perimeter of the tire that comes in contact with the road, so we first find the circumference of the tire:

$$C = 2\pi r = 2\pi(15) = 30\pi \text{ in.}$$

This means that 1 rotation $= 30\pi$ in. From this point we proceed by converting "miles per hour" to "rotations per second" by a series of **conversion factors** that are really factors of 1:

$$\frac{70 \text{ miles}}{1 \text{ hour}} \times \frac{1 \text{ hour}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{1 \text{ rotation}}{30\pi \text{ inches}}$$

$$= \frac{70 \times 5280 \times 12 \text{ rotations}}{60 \times 60 \times 30\pi \text{ sec}} \approx 13.07 \text{ rotations per second}$$

*Now try Exercise 39.*

# Functions from Data

In this course we will use the following 3-step strategy to construct functions from data.

### Constructing a Function from Data

Given a set of data points of the form $(x, y)$, to construct a formula that approximates $y$ as a function of $x$:

1. Make a scatter plot of the data points. The points do not need to pass the vertical line test.
2. Determine from the shape of the plot whether the points seem to follow the graph of a familiar type of function (line, parabola, cubic, sine curve, etc.).
3. Transform a basic function of that type to fit the points as closely as possible.

Step 3 might seem like a lot of work, and for earlier generations it certainly was; it required all of the tricks of Section 1.6 and then some. We, however, will gratefully use technology to do this "curve-fitting" step for us, as shown in Example 6.

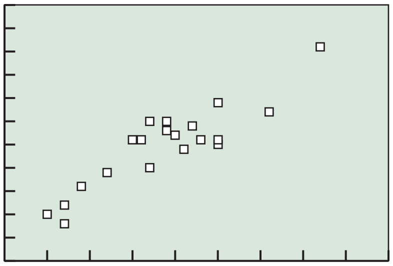



**FIGURE 1.84** The scatter plot of the temperature data in Example 6.

## EXAMPLE 6 Curve-Fitting with Technology

Table 1.12 records the low and high daily temperatures observed on 9/9/1999 in 20 major American cities. Find a function that approximates the high temperature $(y)$ as a function of the low temperature $(x)$. Use this function to predict the high temperature that day for Madison, WI, given that the low was 46.

**SOLUTION** The scatter plot is shown in Figure 1.84.

**Table 1.12 Temperature on 9/9/99**

| City | Low | High | City | Low | High |
|---|---|---|---|---|---|
| New York, NY | 70 | 86 | Miami, FL | 76 | 92 |
| Los Angeles, CA | 62 | 80 | Honolulu, HI | 70 | 85 |
| Chicago, IL | 52 | 72 | Seattle, WA | 50 | 70 |
| Houston, TX | 70 | 94 | Jacksonville, FL | 67 | 89 |
| Philadelphia, PA | 68 | 86 | Baltimore, MD | 64 | 88 |
| Albuquerque, NM | 61 | 86 | St. Louis, MO | 57 | 79 |
| Phoenix, AZ | 82 | 106 | El Paso, TX | 62 | 90 |
| Atlanta, GA | 64 | 90 | Memphis, TN | 60 | 86 |
| Dallas, TX | 65 | 87 | Milwaukee, WI | 52 | 68 |
| Detroit, MI | 54 | 76 | Wilmington, DE | 66 | 84 |

*Source: AccuWeather, Inc.*

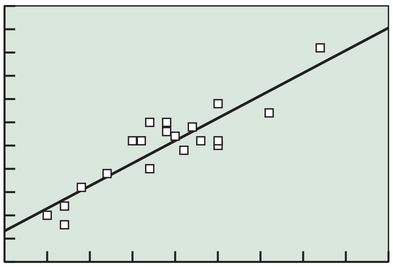



**FIGURE 1.85** The temperature scatter plot with the regression line shown. (Example 6)

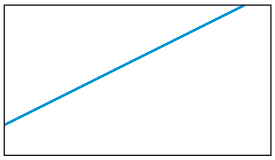
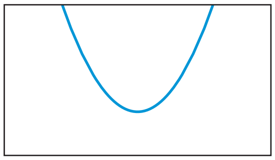
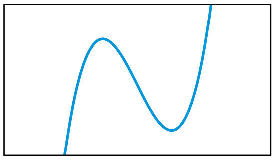
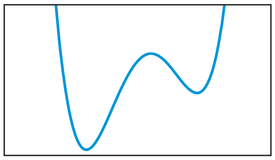
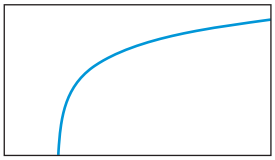
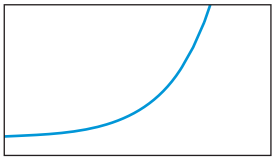
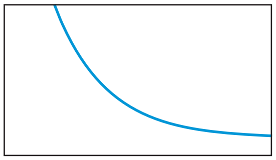
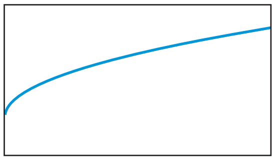
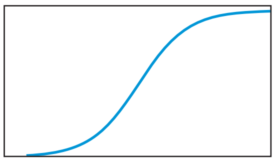
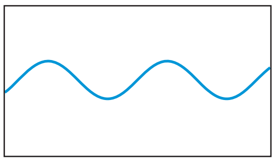
Notice that the points do not fall neatly along a well-known curve, but they do seem to fall *near* an upwardly sloping line. We therefore choose to model the data with a function whose graph is a line. We could fit the line by sight (as we did in Example 5 in Section 1.1), but this time we will use the calculator to find the line of "best fit," called the **regression line**. (See your grapher's owner's manual for how to do this.) The regression line is found to be approximately $y = 0.97x + 23$. As Figure 1.85 shows, the line fits the data as well as can be expected.

If we use this function to predict the high temperature for the day in Madison, WI, we get $y = 0.97(46) + 23 = 67.62$. (For the record, the high that day was 67.)

***Now try Exercise 47, parts (a) and (b).***

Professional statisticians would be quick to point out that this function should not be trusted as a model for all cities, despite the fairly successful prediction for Madison. (For example, the prediction for San Francisco, with a low of 54 and a high of 64, is off by more than 11 degrees.) *The effectiveness of a data-based model is highly dependent on the number of data points and on the way they were selected.* The functions we construct from data in this book should be analyzed for how well they model the data, not for how well they model the larger population from which the data came.

In addition to lines, we can model scatter plots with several other curves by choosing the appropriate regression option on a calculator or computer. The options to which we will refer in this book (and the chapters in which we will study them) are shown in the following table:

| Regression Type | Equation | Graph | Applications |
|---|---|---|---|
| Linear (Chapter 2) | $y = ax + b$ |  | Fixed cost plus variable cost, linear growth, free-fall velocity, simple interest, linear depreciation, many others |
| Quadratic (Chapter 2) | $y = ax^2 + bx + c$ (requires at least 3 points) |  | Position during free fall, projectile motion, parabolic reflectors, area as a function of linear dimension, quadratic growth, etc. |
| Cubic (Chapter 2) | $y = ax^3 + bx^2 + cx + d$ (requires at least 4 points) |  | Volume as a function of linear dimension, cubic growth, miscellaneous applications where quadratic regression does not give a good fit |
| Quartic (Chapter 2) | $y = ax^4 + bx^3 + cx^2 + dx + e$ (requires at least 5 points) |  | Quartic growth, miscellaneous applications where quadratic and cubic regression do not give a good fit |
| Natural logarithmic (ln) (Chapter 3) | $y = a + b \ln x$ (requires $x > 0$) |  | Logarithmic growth, decibels (sound), Richter scale (earthquakes), inverse exponential models |
| Exponential $(b > 1)$ (Chapter 3) | $y = a \cdot b^x$ |  | Exponential growth, compound interest, population models |
| Exponential $(0 < b < 1)$ (Chapter 3) | $y = a \cdot b^x$ |  | Exponential decay, depreciation, temperature loss of a cooling body, etc. |
| Power (requires $x, y > 0$) (Chapter 2) | $y = a \cdot x^b$ |  | Inverse-square laws, Kepler's Third Law |
| Logistic (Chapter 3) | $y = \dfrac{c}{1 + a \cdot e^{-bx}}$ |  | Logistic growth: spread of a rumor, population models |
| Sinusoidal (Chapter 4) | $y = a \sin(bx + c) + d$ |  | Periodic behavior: harmonic motion, waves, circular motion, etc. |

### Displaying Diagnostics

If your calculator is giving regression formulas but not displaying the values of $r$ or $r^2$ or $R^2$, you may be able to fix that. Go to the CATALOG menu and choose a command called "DiagnosticOn." Enter the command on the home screen and see the reply "Done." Your next regression should display the diagnostic values.

These graphs are only examples, as they can vary in shape and orientation. (For example, any of the curves could appear upside-down.) The grapher uses various strategies to fit these curves to the data, most of them based on combining function composition with linear regression. Depending on the regression type, the grapher may display a number $r$ called the **correlation coefficient** or a number $r^2$ or $R^2$ called the **coefficient of determination**. In either case, a useful "rule of thumb" is that *the closer the absolute value of this number is to 1, the better the curve fits the data.*

We can use this fact to help choose a regression type, as in Exploration 1.

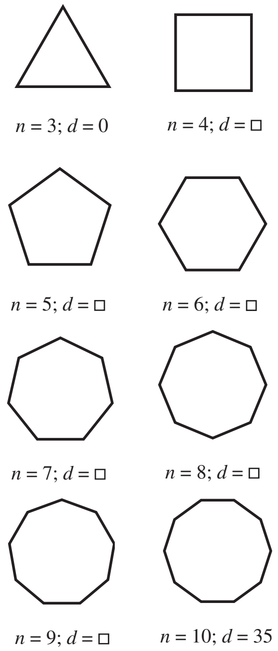



**FIGURE 1.86** Some polygons. (Exploration 1)

### EXPLORATION 1 Diagonals of a Regular Polygon

How many diagonals does a regular polygon have? Can the number be expressed as a function of the number of sides? Try this Exploration.

1. Draw in all the diagonals (i.e., segments connecting nonadjacent points) in each of the regular polygons shown and fill in the number ($d$) of diagonals in the space below the figure. The values of $d$ for the triangle $(n = 3)$ and the decagon $(n = 10)$ are filled in for you.
2. Put the values of $n$ in list L1, *beginning with* $n = 4$. (We want to avoid that $y = 0$ value for some of our regressions later.) Put the corresponding values of $d$ in list L2. Display a scatter plot of the ordered pairs.
3. The graph shows an increasing function with some curvature, but it is not clear which kind of growth would fit it best. Try these regressions (preferably in the given order) and record the value of $r^2$ or $R^2$ for each: linear, power, quadratic, cubic, quartic. (Note that the curvature is not right for logarithmic, logistic, or sinusoidal curve-fitting, so it is not worth it to try those.)
4. What kind of curve is the best fit? (It might appear at first that there is a tie, but look more closely at the functions you get.) How good *is* the fit?
5. Looking back, could you have predicted the results of the cubic and quartic regressions after seeing the result of the quadratic regression?
6. The "best-fit" curve gives the actual formula for $d$ as a function of $n$. (In Chapter 9 you will learn how to derive this formula for yourself.) Use the formula to find the number of diagonals of a 128-gon.

We will have more to say about curve fitting as we study the various function types in later chapters.

## Chapter Opener Problem (from page 63)

**Problem:** The table below shows the growth in the Consumer Price Index (CPI) for housing for selected years between 1990 and 2007 (based on 1983 dollars). How can we construct a function to predict the housing CPI for the years 2008–2015?

**Consumer Price Index (Housing)**

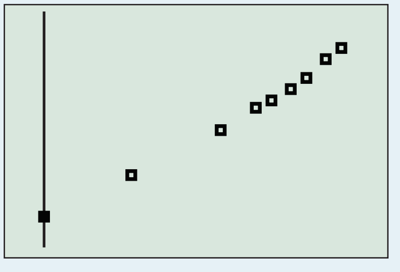
| Year | Housing CPI |
|---|---|
| 1990 | 128.5 |
| 1995 | 148.5 |
| 2000 | 169.6 |
| 2002 | 180.3 |
| 2003 | 184.8 |
| 2004 | 189.5 |
| 2005 | 195.7 |
| 2006 | 203.2 |
| 2007 | 209.6 |

*Source: Bureau of Labor Statistics, quoted in The World Almanac and Book of Facts 2009.*

**Solution:** A scatter plot of the data is shown in Figure 1.87, where $x$ is the number of years since 1990. A linear model would work pretty well, but the slight upward curve of the scatter plot suggests that a quadratic model might work better. Using a calculator to compute the quadratic regression curve, we find its equation to be

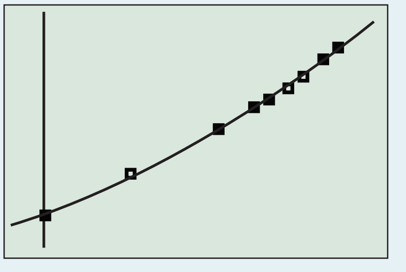
$$y = 0.089x^2 + 3.17x + 129.$$

As Figure 1.88 shows, the parabola fits the data impressively well.



[–2, 19] by [115, 225]

**FIGURE 1.87** Scatter plot of the data for the housing CPI.



[–2, 19] by [115, 225]

**FIGURE 1.88** Scatter plot with the regression curve shown.

To predict the housing CPI for 2008, use $x = 18$ in the regression equation. Similarly, we can predict the housing CPI for each of the years 2008–2015, as shown below:

**Predicted CPI (Housing)**

| Year | Predicted Housing CPI |
|---|---|
| 2008 | 214.9 |
| 2009 | 221.4 |
| 2010 | 228.0 |
| 2011 | 234.8 |
| 2012 | 241.8 |
| 2013 | 249.0 |
| 2014 | 256.3 |
| 2015 | 263.9 |

Even with a regression curve that fits the data as beautifully as in Figure 1.88, it is risky to predict this far beyond the data set. Statistics like the CPI are dependent on many volatile factors that can quickly render any mathematical model obsolete. In fact, the mortgage model that fueled the housing growth up to 2007 proved to be unsustainable, and when it broke down it took many well-behaved economic curves (like this one) down with it. In light of that fact, you might enjoy comparing these "predictions" with the actual housing CPI numbers as the years go by!

## QUICK REVIEW 1.7 *(For help, go to Section P.3 and P.4.)*

In Exercises 1–10, solve the given formula for the given variable.

1. **Area of a Triangle** Solve for $h$: $A = \frac{1}{2}bh$
2. **Area of a Trapezoid** Solve for $h$: $A = \frac{1}{2}(b_1 + b_2)h$
3. **Volume of a Right Circular Cylinder** Solve for $h$: $V = \pi r^2 h$
4. **Volume of a Right Circular Cone** Solve for $h$: $V = \frac{1}{3}\pi r^2 h$
5. **Volume of a Sphere** Solve for $r$: $V = \frac{4}{3}\pi r^3$
6. **Surface Area of a Sphere** Solve for $r$: $A = 4\pi r^2$
7. **Surface Area of a Right Circular Cylinder** Solve for $h$: $A = 2\pi rh + 2\pi r^2$
8. **Simple Interest** Solve for $t$: $I = Prt$
9. **Compound Interest** Solve for $P$: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
10. **Free-Fall from Height *H*** Solve for $t$: $s = H - \frac{1}{2}gt^2$

## SECTION 1.7 EXERCISES

**Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator.***

In Exercises 1–10, write a mathematical expression for the quantity described verbally.

1. Five more than three times a number $x$
2. A number $x$ increased by 5 and then tripled
3. Seventeen percent of a number $x$
4. Four more than 5% of a number $x$
5. **Area of a Rectangle** The area of a rectangle whose length is 12 more than its width $x$
6. **Area of a Triangle** The area of a triangle whose altitude is 2 more than its base length $x$
7. **Salary Increase** A salary after a 4.5% increase, if the original salary is $x$ dollars
8. **Income Loss** Income after a 3% drop in the current income of $x$ dollars
9. **Sale Price** Sale price of an item marked $x$ dollars, if 40% is discounted from the marked price
10. **Including Tax** Actual cost of an item selling for $x$ dollars if the sales tax rate is 8.75%

In Exercises 11–14, choose a variable and write a mathematical expression for the quantity described verbally.

11. **Total Cost** The total cost is $34,500 plus $5.75 for each item produced.
12. **Total Cost** The total cost is $28,000 increased by 9% plus $19.85 for each item produced.
13. **Revenue** The revenue when each item sells for $3.75
14. **Profit** The profit consists of a franchise fee of $200,000 plus 12% of all sales.

In Exercises 15–20, write the specified quantity as a function of the specified variable. It will help in each case to draw a picture.

15. The height of a right circular cylinder equals its diameter. Write the volume of the cylinder as a function of its radius.
16. One leg of a right triangle is twice as long as the other. Write the length of the hypotenuse as a function of the length of the shorter leg.
17. The base of an isosceles triangle is half as long as the two equal sides. Write the area of the triangle as a function of the length of the base.
18. A square is inscribed in a circle. Write the area of the square as a function of the radius of the circle.
19. A sphere is contained in a cube, tangent to all six faces. Find the surface area of the cube as a function of the radius of the sphere.
20. An isosceles triangle has its base along the $x$-axis with one base vertex at the origin and its vertex in the first quadrant on the graph of $y = 6 - x^2$. Write the area of the triangle as a function of the length of the base.

In Exercises 21–36, write an equation for the problem and solve the problem.

21. One positive number is 4 times another positive number. The sum of the two numbers is 620. Find the two numbers.

**22.** When a number is added to its double and its triple, the sum is 714. Find the three numbers.

**23. Salary Increase** Mark received a 3.5% salary increase. His salary after the raise was $36,432. What was his salary before the raise?

**24. Consumer Price Index** The Consumer Price Index for food and beverages in 2007 was 203.3 after a hefty 3.9% increase from the previous year. What was the Consumer Price Index for food and beverages in 2006? (Source: *U.S. Bureau of Labor Statistics*)

**25. Travel Time** A traveler averaged 52 miles per hour on a 182-mile trip. How many hours were spent on the trip?

**26. Travel Time** On their 560-mile trip, the Bruins basketball team spent two more hours on the interstate highway than they did on local highways. They averaged 45 mph on local highways and 55 mph on the interstate highways. How many hours did they spend driving on local highways?

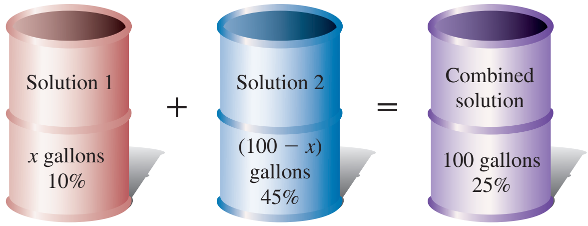
**27. Sale Prices** At a shirt sale, Jackson sees two shirts that he likes equally well. Which is the better bargain, and why?


**28. Job Offers** Ruth is weighing two job offers from the sales departments of two competing companies. One offers a base salary of $25,000 plus 5% of gross sales; the other offers a base salary of $20,000 plus 7% of gross sales. What would Ruth's gross sales total need to be to make the second job offer more attractive than the first?

**29. Cell Phone Antennas** From December 2006 to December 2007, the number of cell phone antennas in the United States grew from 195,613 to 213,299. What was the percentage increase in U.S. cell phone antennas in that one-year period? (*Source: CTIA, quoted in The World Almanac and Book of Facts 2009*)

**30. Cell phone Antennas** From December 1996 to December 1997, the number of cell phone antennas in the United States grew from 30,045 to 51,600. What was the percentage increase in U.S. cell phone antennas in that one-year period? (*Source: CTIA, quoted in The World Almanac and Book of Facts 2009*)

**31. Mixing Solutions** How much 10% solution and how much 45% solution should be mixed together to make 100 gallons of 25% solution?




**(a)** Write an equation that models this problem.

**(b)** Solve the equation graphically.

**32. Mixing Solutions** The chemistry lab at the University of Hardwoods keeps two acid solutions on hand. One is 20% acid and the other is 35% acid. How much 20% acid solution and how much 35% acid solution should be used to prepare 25 liters of a 26% acid solution?

**33. Maximum Value Problem** A square of side $x$ inches is cut out of each corner of a 10 in. by 18 in. piece of cardboard and the sides are folded up to form an open-topped box.

**(a)** Write the volume $V$ of the box as a function of $x$.

**(b)** Find the domain of your function, taking into account the restrictions that the model imposes in $x$.

**(c)** Use your graphing calculator to determine the dimensions of the cut-out squares that will produce the box of maximum volume.

**34. Residential Construction** DDL Construction is building a rectangular house that is 16 feet longer than it is wide. A rain gutter is to be installed in four sections around the 136-foot perimeter of the house. What lengths should be cut for the four sections?

**35. Protecting an Antenna** In Example 3, suppose the parabolic dish has a 32-in. diameter and is 8 in. deep, and the radius of the cardboard cylinder is 8 in. Now how tall must the cylinder be to fit in the middle of the dish and be flush with the top of the dish?

**36. Interior Design** Renée's Decorating Service recommends putting a border around the top of the four walls in a dining room that is 3 feet longer than it is wide. Find the dimensions of the room if the total length of the border is 54 feet.

**37. Finding the Model and Solving** Water is stored in a conical tank with a faucet at the bottom. The tank has depth 24 in. and radius 9 in., and it is filled to the brim. If the faucet is opened to allow the water to flow at a rate of 5 cubic inches per second, what will the depth of the water be after 2 minutes?

**38. Investment Returns** Reggie invests $12,000, part at 7% annual interest and part at 8.5% annual interest. How much is invested at each rate if Reggie's total annual interest is $900?

**39. Unit Conversion** A tire of a moving bicycle has radius 16 in. If the tire is making 2 rotations per second, find the bicycle's speed in miles per hour.

**40. Investment Returns** Jackie invests $25,000, part at 5.5% annual interest and the balance at 8.3% annual interest. How much is invested at each rate if Jackie receives a 1-year interest payment of $1571?

## Standardized Test Questions

**41. True or False** A correlation coefficient gives an indication of how closely a regression line or curve fits a set of data. Justify your answer.

**42. True or False** Linear regression is useful for modeling the position of an object in free fall. Justify your answer.

In Exercises 43–46, tell which type of regression is likely to give the most accurate model for the scatter plot shown without using a calculator.

**(A)** Linear regression

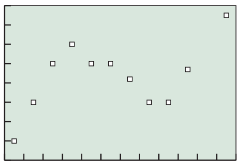
**(B)** Quadratic regression

**(C)** Cubic regression

**(D)** Exponential regression

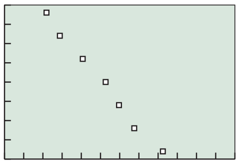
**(E)** Sinusoidal regression

**43. Multiple Choice**



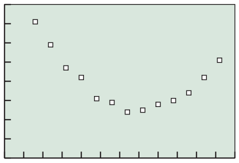
[0, 12] by [0, 8]

**44. Multiple Choice**



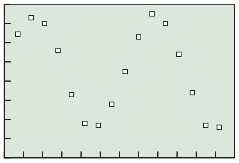
[0, 12] by [0, 8]

**45. Multiple Choice**



[0, 12] by [0, 8]

**46. Multiple Choice**



[0, 12] by [0, 8]

## Exploration

**47. Manufacturing** The Buster Green Shoe Company determines that the annual cost $C$ of making $x$ pairs of one type of shoe is $30 per pair plus $100,000 in fixed overhead costs. Each pair of shoes that is manufactured is sold wholesale for $50.

**(a)** Find an equation that models the cost of producing $x$ pairs of shoes.

**(b)** Find an equation that models the revenue produced from selling $x$ pairs of shoes.

**(c)** Find how many pairs of shoes must be made and sold in order to break even.

**(d)** Graph the equations in (a) and (b). What is the graphical interpretation of the answer in (c)?

**48. Employee Benefits** John's company issues employees a contract that identifies salary and the company's contributions to pension, health insurance premiums, and disability insurance. The company uses the following formulas to calculate these values.

| Salary | $x$ (dollars) |
|---|---|
| Pension | 12% of salary |
| Health insurance | 3% of salary |
| Disability insurance | 0.4% of salary |

If John's total contract with benefits is worth $48,814.20, what is his salary?

**49. Manufacturing** Queen, Inc., a tennis racket manufacturer, determines that the annual cost $C$ of making $x$ rackets is $23 per racket plus $125,000 in fixed overhead costs. It costs the company $8 to string a racket.

**(a)** Find a function $y_1 = u(x)$ that models the cost of producing $x$ unstrung rackets.

**(b)** Find a function $y_2 = s(x)$ that models the cost of producing $x$ strung rackets.


**(c)** Find a function $y_3 = R_u(x)$ that models the revenue generated by selling $x$ unstrung rackets.

**(d)** Find a function $y_4 = R_s(x)$ that models the revenue generated by selling $x$ rackets.

**(e)** Graph $y_1$, $y_2$, $y_3$, and $y_4$ simultaneously in the window [0, 10,000] by [0, 500,000].

**(f) Writing to Learn** Write a report to the company recommending how they should manufacture their rackets, strung or unstrung. Assume that you can include the viewing window in (e) as a graph in the report, and use it to support your recommendation.

**50. Hourly Earnings of U.S. Production Workers** The average hourly earnings of U.S. production workers for 1990–2007 are shown in Table 1.13.

**Table 1.13 Average Hourly Earnings**

| Year | Average Hourly Earnings ($) |
|---|---|
| 1990 | 10.20 |
| 1991 | 10.52 |
| 1992 | 10.77 |
| 1993 | 11.05 |
| 1994 | 11.34 |
| 1995 | 11.65 |
| 1996 | 12.04 |
| 1997 | 12.51 |
| 1998 | 13.01 |
| 1999 | 13.49 |
| 2000 | 14.02 |
| 2001 | 14.54 |
| 2002 | 14.97 |
| 2003 | 15.37 |
| 2004 | 15.69 |
| 2005 | 16.13 |
| 2006 | 16.76 |
| 2007 | 17.42 |

*Source: Bureau of Labor Statistics as quoted in The World Almanac and Book of Facts 2009.*

**(a)** Produce a scatter plot of the hourly earnings $(y)$ as a function of years since 1990 $(x)$.

**(b)** Find the linear regression equation for the years 1990–1998. Round the coefficients to the nearest 0.001.

**(c)** Find the linear regression equation for the years 1990–2007. Round the coefficients to the nearest 0.001.

**(d)** Use both lines to predict the hourly earnings for the year 2010. How different are the estimates? Which do you think is a safer prediction of the true value?

**(e)** **Writing to Learn** Use the results of parts (a)–(d) to explain why it is risky to predict $y$-values for $x$-values that are not very close to the data points, even when the regression plot fits the data points quite well.

## Extending the Ideas

**51. Newton's Law of Cooling** A 190° cup of coffee is placed on a desk in a 72° room. According to Newton's Law of Cooling, the temperature $T$ of the coffee after $t$ minutes will be $T = (190 - 72)b^t + 72$, where $b$ is a constant that depends on how easily the cooling substance loses heat. The data in Table 1.14 are from a simulated experiment of gathering temperature readings from a cup of coffee in a 72° room at 20 one-minute intervals:

**Table 1.14 Cooling a Cup of Coffee**

| Time | Temp | Time | Temp |
|---|---|---|---|
| 1 | 184.3 | 11 | 140.0 |
| 2 | 178.5 | 12 | 136.1 |
| 3 | 173.5 | 13 | 133.5 |
| 4 | 168.6 | 14 | 130.5 |
| 5 | 164.0 | 15 | 127.9 |
| 6 | 159.2 | 16 | 125.0 |
| 7 | 155.1 | 17 | 122.8 |
| 8 | 151.8 | 18 | 119.9 |
| 9 | 147.0 | 19 | 117.2 |
| 10 | 143.7 | 20 | 115.2 |

**(a)** Make a scatter plot of the data, with the times in list L1 and the temperatures in list L2.

**(b)** Store L2 − 72 in list L3. The values in L3 should now be an exponential function $(y = a \times b^x)$ of the values in L1.

**(c)** Find the exponential regression equation for L3 as a function of L1. How well does it fit the data?

**52. Group Activity Newton's Law of Cooling** If you have access to laboratory equipment (such as a CBL or CBR unit for your grapher), gather experimental data such as in Exercise 51 from a cooling cup of coffee. Proceed as follows:

**(a)** First, use the temperature probe to record the temperature of the room. It is a good idea to turn off fans and air conditioners that might affect the temperature of the room during the experiment. It should be a constant.

**(b)** Heat the coffee. It need not be boiling, but it should be at least 160°. (It also need not be coffee.)

**(c)** Make a new list consisting of the temperature values minus the room temperature. Make a scatter plot of this list $(y)$ against the time values $(x)$. It should appear to approach the $x$-axis as an asymptote.

**(d)** Find the equation of the exponential regression curve. How well does it fit the data?

**(e)** What is the equation predicted by Newton's Law of Cooling? (Substitute your initial coffee temperature and the temperature of your room for the 190 and 72 in the equation in Exercise 51.)

**(f)** **Group Discussion** What sort of factors would affect the value of $b$ in Newton's Law of Cooling? Discuss your ideas with the group.