Name Key Date 9/3 - 9/6 Period______

Worksheet 2.1—Tangent Line Problem

Show all work. No calculator permitted, except when stated.

Short Answer

1. Find the derivative function, $f'(x)$, for each of the following using the limit definition.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

(a) $f(x)=2x^2+3x-4$

$$f'(x)=\lim_{h\to 0}\frac{2(x+h)^2+3(x+h)-4-2x^2-3x+4}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{2x^2}+4xh+2h^2+\cancel{3x}+3h\cancel{-4}\cancel{-2x^2}\cancel{-3x}\cancel{+4}}{h}$$

$$\lim_{h\to 0}\frac{\cancel{h}(4x+2h+3)}{\cancel{h}} = 4x+3$$

(b) $f(x)=\dfrac{3}{x-1}$

$$f'(x)=\lim_{h\to 0}\frac{\frac{3}{x+h-1}-\frac{3}{x-1}\cdot\frac{(x-1)(x+h-1)}{(x-1)(x+h-1)}}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{3(x-1)-3(x+h-1)}{h(x-1)(x+h-1)}$$

$$f'(x)=\lim_{h\to 0}\frac{\cancel{3x}\cancel{-3}\cancel{-3x}-3h\cancel{+3}}{h(x-1)(x+h-1)}$$

$$=\lim_{h\to 0}\frac{-3\cancel{h}}{\cancel{h}((x-1)(x+h)-1)}=\frac{-3}{(x-1)^2}$$

(c) $f(x)=\sqrt{x-2}$

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h-2}-\sqrt{x-2}}{h}$$

$$\lim_{h\to 0}\frac{x+h\cancel{-2}\cancel{-x}\cancel{+2}}{h(\sqrt{x+h-2}+\sqrt{x-2})}$$

$$\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-2}+\sqrt{x-2})}=\frac{1}{2\sqrt{x-2}}$$

2. Find the slope of the tangent lines to the graphs of the following functions at the indicated points. Use the *alternate form.*

$$f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

(a) $f(x)=3-2x$ at $(-1,5)$

$$f'(x)=\lim_{x\to -1}\frac{3-2x-5}{x+1}$$

$$=\lim_{x\to -1}\frac{-2x-2}{x+1}$$

$$=\lim_{x\to -1}\frac{-2\cancel{(x+1)}}{\cancel{(x+1)}}$$

$$=-2$$

(b) $g(x)=5-x^2$ at $x=2$ $\quad g(2)=1$

$$g'(x)=\lim_{x\to 2}\frac{5-x^2-1}{x-2}$$

$$=\lim_{x\to 2}\frac{-x^2+4}{x-2}$$

$$=\lim_{x\to 2}\frac{-(x+2)\cancel{(x-2)}}{\cancel{x-2}}$$

$$=-4$$

3. Find the equation of the tangent line, in Taylor Form: $y = y_1 + m(x - x_1)$, for $g(x) = x^2 + 1$ at $(2,5)$. Use the ***modified form*** to find $g'(2)$.

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

① $g'(2) =$

$\lim_{h \to 0} \frac{(2+h)^2 + 1 - 5}{h}$

$\lim_{h \to 0} \frac{\cancel{4} + 4h + h^2 - \cancel{4}}{h}$

$\lim_{h \to 0} \frac{\cancel{h}(4+h)}{\cancel{h}} = 4$

② Need a point $(2,5)$

③ $y = 5 + 4(x - 2)$

4. Find the equation of the tangent line, in Taylor Form: $y = y_1 + m(x - x_1)$, for $y = \sqrt{x} - 1$ at $c = 9$. Use the ***alternate form*** to find $y'(9)$.

$y(9) = 2$

$$y'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

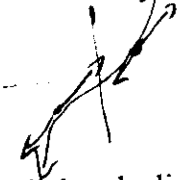
① $y'(9) = \lim_{x \to 9} \frac{\sqrt{x} - 1 - 2}{x - 9} = \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \frac{(\sqrt{x} + 3)}{(\sqrt{x} + 3)}$

$= \lim_{x \to 9} \frac{\cancel{x - 9}}{\cancel{(x - 9)}(\sqrt{x} + 3)}$

$= \frac{1}{6}$

② pt $(9, 2)$

③ $y = 2 + \frac{1}{6}(x - 9)$



5. Find an equation of the line that is tangent to $f(x) = x^3$ and parallel to the line $3x - y + 1 = 0$. Remember, parallel lines have the same slope, but different base camps.

① $m = 3 \qquad 3x + 1 = y$

so, $f'(c) = 3$

② Need a point

$$f'(c) = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

Remember $f'(c) = 3$

$$f'(c) = \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$3 = 3x^2 \rightarrow x^2 = 1 \quad x = \pm 1$$

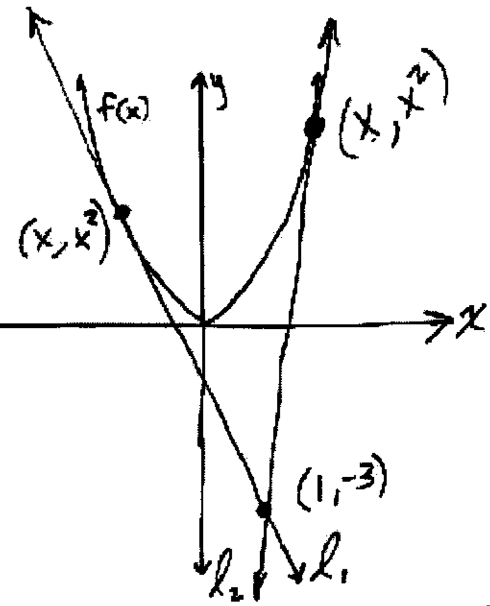
③ for $x = 1 \rightarrow f(1) = 1$

$y = 1 + 3(x - 1)$

for $x = -1 \rightarrow f(-1) = -1$

$y = -1 + 3(x + 1)$

6. Find the equations of the two lines, ℓ_1 and ℓ_2, that are tangent to the graph of $f(x) = x^2$ if each pass through the point $(1, -3)$, as shown at right. Hint: equate two different expressions for finding the slope of a line. Solve the resulting equation.



①

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x + h)}{\cancel{h}}$$

$$= 2x$$

expression

$$f'(x) = \frac{x^2 + 3}{x - 1}$$

second

② $\dfrac{x^2 + 3}{x - 1} = 2x$

$x^2 + 3 = 2x(x - 1)$

$x^2 + 3 = 2x^2 - 2x$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x = 3, \; x = -1$

③ Find pts.

a) $(-1, 1) \rightarrow f'(-1) = 2$

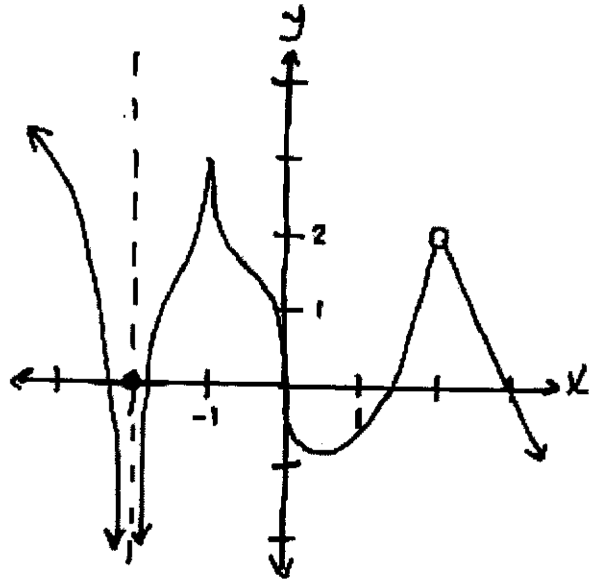
b) $(3, 9) \rightarrow f'(3) = 6$

④ $(-1, 1)$, $m = -2$

$y = 1 + -2(x + 1)$

$(3, 9)$, $m = 6$

$y = 9 + 6(x - 3)$



7. The graph of a function $f(x)$ is show above. For which value(s) of x is the graph of $f(x)$ not differentiable. In each case, explain why not.

f(x) is not differentiable at:

$x = -2$ → discontinuity

$x = -1$ → cusp

$x = 0$ → vertical tangent

$x = 2$ → discontinuity

8. For each of the following, the limit represents $f'(c)$ for a function $f(x)$ and a number $x = c$. Find both f and c.

(a) $\lim_{h \to 0} \frac{[5-3(1+h)]-2}{h}$

$f(x) = 5 - 3x$

$c = 1$

(b) $\lim_{h \to 0} \frac{(-2+h)^3+8}{h}$

$f(x) = x^3$

$c = -2$

(c) $\lim_{x \to 6} \frac{-x^2+36}{x-6}$

$f(x) = -x^2$

$c = 6$

(d) $\lim_{x \to 9} \frac{2\sqrt{x}-6}{x-9}$

$f(x) = 2\sqrt{x}$

$c = 9$

$f'(x) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

9. Using the alternate form, determine whether each of the following function is differentiable at the indicated point. Show the work that leads to your answer.

$f(0) = 5$

(a) $f(x) = \begin{cases} 5 - 4x, & x \le 0 \\ -2x^2, & x > 0 \end{cases}$ at $x = 0$

$f'(0) = \lim_{x \to 0^-} \frac{\cancel{5} - 4\cancel{x} - \cancel{5}}{\cancel{x} - 0} = -4$

$\lim_{x \to 0^+} \frac{-2x^2 - 5}{x - 0} = \frac{\#}{0} \to$ dne

$\boxed{f(x) \text{ is not diff at } x = 0 \text{ since } \lim_{x \to 0^-} f(x) \ne \lim_{x \to 0^+} f(x)}$

$f(1) = 6$

(b) $f(x) = \begin{cases} (x-1)^3, & x \le 1 \\ (x-1)^2, & x > 1 \end{cases}$ at $x = 1$

$\lim_{x \to 1^-} \frac{(x-1)^3}{x-1} \to 0 = \lim_{x \to 1^-} \frac{(x-1)(x^2+x+1) - 0}{(x-1)} = 3$

$\lim_{x \to 1^+} \frac{(x-1)^2}{x-1} = 2$

$f(x)$ is not diff at $x = 1$ since $\lim_{x \to 1^-} f(x) \ne \lim_{x \to 1^+} f(x)$

10. **True or False. If false, explain why or give a counterexample.**

(a) The slope of the tangent line to the differentiable function f at the point $(2, f(2))$ is $\frac{f(2+h) - f(2)}{h}$.

False, this is the slope of the secant line

(b) If a function is continuous at a point, then that function is differentiable at that point.

No $f(x) = |x|$ at $x = 0$

(c) If a function's slopes from both the right and the left at a point are the same, then that function is differentiable at that point.

false, $f(x) = \frac{x^2 - 4}{(x-2)}$

(d) If a function is differentiable at a point, then that function is continuous at that point.

True $D \to C$

11. Using your **calculator** to zoooooooom in, determine if $h(x)=\sqrt{x^2+0.0001}+0.99$ is locally linear at $x=0$. Give a reason for your answer.

Multiple Choice

E 12. A function will fail to be differentiable at all of the following except

(A) A vertical asymptote (B) A removable discontinuity (C) A cusp
(D) A vertical tangent line (E) A horizontal tangent line

$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2}, & x\neq 2\\ 1, & x=2\end{cases}$$

$\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

A 13. Let f be the function defined above. Which of the following statements about f are true?

I. $\lim\limits_{x\to 2} f(x)$ exists ✓
II. f is continuous at $x=2$ ✗
III. f is differentiable at $x=2$ ✗

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

I.
$\lim\limits_{x\to 2^-} f(x)=4$
$\lim\limits_{x\to 2^+} f(x)=4$

II.
$f(2)=1$

B 14. Let f be a differentiable function such that $f(2)=1$ and $f'(2)=4$. Let $T(x)$ be the equation of the tangent line to $f(x)$ at $x=2$. What is the value of $T(1.9)$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

pt $(2,1)$
$m=4$ at $x=2$

$T(x)=1+4(x-2)$
$=1+4(-.1)=.6$

C 15. Let f be a function such that $\lim_{h\to 0}\frac{f(7+h)-f(7)}{h}=5$. Which of the followign must be true?

I. f is continuous at $x=7$ ✓
II. f is differentiable at $x=7$ ✓
III. The derivative of f is differentiable at $x=7$? Not sure abt what f(x) is to determine if its der. is diff

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

B 16. At $x=4$, the function given by $h(x)=\begin{cases} x^2, & x\le 4 \\ 4x, & x>4 \end{cases}$ is

(A) Undefined
(B) Continuous but not differentiable
(C) Differentiable but not continuous
(D) Neither continuous nor differentiable
(E) Both continuous and differentiable

Not diff

$$\lim_{x\to 4^-}\frac{x^2-16}{x-4}=8$$

$$\lim_{x\to 4^+}\frac{4x-16}{x-4}=4$$

Continuous

$$\lim_{x\to 4^-} h(x)=16$$

$$\lim_{x\to 4^+} h(x)=16$$

$f(4)=16$

So continuous