4.4 Solving Systems with Matrices

$\boxed{\text{ex1}}$ Solve $\begin{cases} -3x + 6y - 4z = 8 \\ x - 4y + 2z = -3 \\ 8y - z = 0 \end{cases}$

Write as matrix equation

$$\underbrace{\begin{bmatrix} -3 & 6 & -4 \\ 1 & -4 & 2 \\ 0 & 8 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 8 \\ -3 \\ 0 \end{bmatrix}}_{B}$$

$3\times 3 \quad 3\times 1$

$$\cancel{A^{-1}}(A)(X) = A^{-1}(B)$$

$$X = (A^{-1})(B)$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 6 & -4 \\ 1 & -4 & 2 \\ 0 & 8 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 8 \\ -3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1.8 \\ -.1 \\ -.8 \end{bmatrix}$$

$$\boxed{(-1.8, -.1, -.8)}$$

$\boxed{\text{ex2}}$ Solve $\begin{cases} 9x - 3y = 27 \\ -6x + 2y = 18 \end{cases}$

$$\begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 27 \\ 18 \end{bmatrix}$$

$$(A)(X) = B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 27 \\ 18 \end{bmatrix} \quad \text{error!}$$

d.n.e

$\det(A) = 18 - 18 = 0 \qquad A^{-1}$ d.n.e

There is no <u>unique solution</u> for the system, it either has infinitely many sol. or no sol.

$$9x - 3y = 27 \qquad\qquad -6x + 2y = 18$$

$$\frac{-3y}{-3} = \frac{-9x}{-3} + \frac{27}{-3} \qquad\qquad \frac{2y}{2} = \frac{6x}{2} + \frac{18}{2}$$

$$y = 3x - 9 \qquad\qquad y = 3x + 9$$

Parallel lines, $\emptyset$

<u>Cramer's Rule</u>: handy way to solve for one variable w/out having to solve the whole system.

$\boxed{\text{ex 2}}$

$$\begin{cases} 2x + y + z = 3 \\ x - y - z = 0 \\ x + 2y + z = 0 \end{cases} \qquad \begin{array}{c} \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix} \\ x \quad y \quad z \end{array} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

<u>1) Solve for x only</u>

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix} \begin{matrix} 2 & 1 \\ 1 & -1 \\ 1 & 2 \end{matrix}} = \frac{3\begin{vmatrix} -1 & -1 \\ 2 & 1 \end{vmatrix}}{(-2 + -1 + 2) - (-1 + -4 + 1)} = \frac{3(-1+2)}{-1-(-4)} = \frac{3}{3} = \boxed{1}$$

<u>2) Solve for y</u>

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix}} = \frac{-3\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}}{3} = \frac{-3(1+1)}{+3} = -\frac{6}{3} = \boxed{-2}$$

<u>3) Solve for z</u>

Determine $f(4)$ given that $f(x)$ is a polynomial of degree 2 where $f(1) = 2$, $f(2) = 3$, $f(3) = 5$

$$\underbrace{f(x)}_{y} = ax^2 + bx + c$$

$$2 = a(1)^2 + b(1) + c \Rightarrow a + b + c = 2$$

$$3 = a(2)^2 + b(2) + c \Rightarrow 4a + 2b + c = 3$$

$$5 = a(3)^2 + b(3) + c \Rightarrow 9a + 3b + c = 5$$

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} a \\ b \\ c \end{bmatrix}} = \underset{B}{\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}}$$

$$A^{-1}\cancel{(A)}(X) = A^{-1}B$$

$$X = (A^{-1})(B)$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} .5 \\ -.5 \\ 2 \end{bmatrix}$$

$$f(x) = ax^2 + bx + c$$

$$f(x) = \frac{1}{2}x^2 - \frac{1}{2}x + 2$$

$$f(4) = \frac{1}{2}(4)^2 - \frac{1}{2}(4) + 2$$

$$= 8 - 2 + 2$$

$$\boxed{f(4) = 8}$$

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17-27 odd, 43.45