4.5 Row-Reduction Method

- a way of solving a system to determine whether it is independent, dependent, or inconsistent.
- performed on an augmented matrix, consisting of coefficients and constant terms in the system

ex 1

system

$$\begin{cases} 3a + b + c = 24 \\ b + 2c = 8 \\ a + 2b + c = 16 \end{cases}$$

Augmented matrix

$$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 24 \\ 0 & 1 & 2 & 8 \\ 1 & 2 & 1 & 16 \end{array}\right]$$

coefficients | constants

Goal of row reduction: transform coefficients col. into an identity matrix by performing Elementary Row operations.

1) Interchange two rows
2) Multiply one row by a nonzero #.
3) Add a multiple of one row to another row

$$\left[\begin{array}{ccc|c} \textcircled{3} & 1 & 1 & 24 \\ 0 & 1 & 2 & 8 \\ 1 & 2 & 1 & 16 \end{array}\right] \quad R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 16 \\ 0 & 1 & 2 & 8 \\ \textcircled{3} & 1 & 1 & 24 \end{array}\right] \quad -3R_1 + R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 16 \\ 0 & \textcircled{1} & 2 & 8 \\ 0 & \textcircled{-5} & -2 & -24 \end{array}\right] \quad 5R_2 + R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 16 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & \textcircled{8} & 16 \end{array}\right] \quad \frac{1}{8}R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 16 \\ 0 & 1 & \textcircled{2} & 8 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad -2R_3 + R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & \textcircled{1} & 16 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad -1R_3 + R_1 \to R_1$$

$$\left[\begin{array}{ccc|c} 1 & \textcircled{2} & 0 & 14 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad -2R_2 + R_1 \to R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$(6, 4, 2)$

ex 2 Solve $\begin{cases} y + z = \frac{3}{2} \\ 2x + 2y + 2z = 4 \\ x + y = 2 \end{cases}$ by row reduction

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & \frac{3}{2} \\ 2 & 2 & 2 & 4 \\ 1 & 1 & 0 & 2 \end{array}\right] \quad \begin{array}{l} R_1 \leftrightarrow R_3 \\ \frac{1}{2} R_2 \to R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & \frac{3}{2} \end{array}\right]$$