KEY

If $|x|=5$ $x=5$ or -5 so $|x|$ ↑ could be (+) + (−)

ASMA Practice November 2015

1. Find all the real values of x which satisfy $|x-1|+|x-2|=7$.

(+) or (−) (+) or (−)

Four cases

① (+)(+)

$(x-1)+(x-2)=7$

$2x-3=7$

$2x=10$

$x=5$

② (+)(−)

$(x-1)+(-x+2)=7$

$x-1-x+2=7$

$1=7$

$\varnothing$

③ (−)(+)

$-x+1+x-2=7$

$-1=7$

$\varnothing$

④ (−)(−)

$-x+1+-x+2=7$

$-2x+3=7$

$-2x=4$

$x=-2$

$x=-2,5$

2. Find all solutions of: $|x-2|=|x+2|-2$.

① (+)(+)

$x-2=x+2-2$

$x-2=x$

$-2=0$

$\varnothing$

② (+)(−)

$x-2=-x-2-2$

$x-2=-x-4$

$2x-2=-4$

$2x=-2$

$x=-1$

extraneous (doesn't work if you plug it back in)

③ (−)(+)

$-x+2=x+2-2$

$-x+2=x$

$2=2x$

$x=1$

④ (−)(−)

$-x+2=-x-2-2$

$-x+2=-x-4$

$2=-4$

$\varnothing$

3. If $2x+3y=2$

$(8x-4z=3)(-2)$

$3y-8z=-1$

$\Rightarrow$ $-16x+8z=-6$

$\Rightarrow$ $(+)\ 3y-8z=-1$

$-16x+3y=-7$

$-16x+3y=-7$

$(-)\ 2x+3y=2$

$-18x=-9$

$\frac{-18x}{-18}=\frac{-9}{-18}$

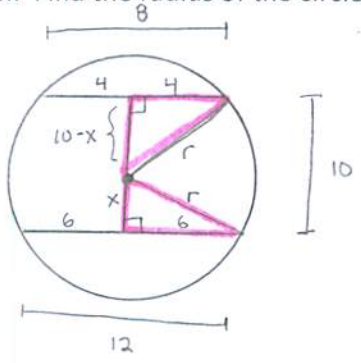
$x=\frac{1}{2}$ Substitute back into other eq. to find y and z

Find the sum $x+y+z$

$z=\frac{1}{4}$ $x=\frac{1}{2}$ $y=\frac{1}{3}$

$x+y+z=\frac{13}{12}$

4. In a circle, chords of lengths 8 and 12 units are parallel, 10 units apart, and the center of the circle is between them. Find the radius of the circle.



pythag thm

$4^2+(10-x)^2=r^2$

$6^2+x^2=r^2$

$4^2+(10-x)^2=6^2+x^2$

$16+(10-x)^2=36+x^2$

FOIL

$16+100-20x+x^2=36+x^2$

$116-20x=36$

$-20x=-80$

$x=4$

$36+(4)^2=r^2$

$36+16=r^2$

$52=r^2$

$r=\sqrt{52}$

$r=\sqrt{4}\cdot\sqrt{13}$

$r=2\sqrt{13}$

5. The least common multiple of 2 positive integers is 36 and their greatest common divisor is 6. Neither number is a multiple of the other. What are the two numbers?

If greatest common divisor is 6, then the #s are divisible by 6. they must be multiples of 6.

multiples of 6: 6, 12, 18, 24

LCM must be 36

multiples:
- 12 — 24 — (36) — 48 ← too big
- 18 — (36)
- 24
- 30

The #s are 12 and 18

6. Find all ordered pairs of real numbers (x, y) for which $x^2 + 4y^2 = 4x - 8y - 8$.

$(x^2 - 4x) + (4y^2 + 8y) = -8$ eq. of circle $(x-h)^2 + (y-k)^2 = r^2$

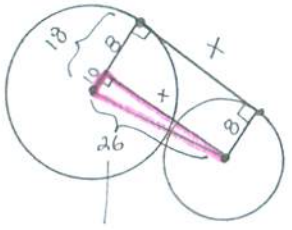
$(x^2 - 4x + \boxed{(-2)^2}) + 4(y^2 + 2y + \boxed{(1)^2}) = -8 + \boxed{4} + \boxed{4}$ complete the square

$(x-2)^2 + 4(y+1)^2 = 0$

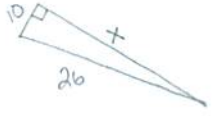
This equation would be true only if $x = 2$ and $y = -1$

$\boxed{(2, -1)}$

7. Two circles of radii 18 and 8 are externally tangent. Find the length of their common external tangent.



Combination of 2 radii (18+8)



$x^2 + 10^2 = 26^2$

$x^2 = 26^2 - 10^2$

$x^2 = 576$

$x = 24$

8. Four rectangles, each 1 cm x 2 cm, are to be packed into a rectangle 2 cm x 4 cm without overlapping. How many distinct arrangements of the smaller rectangles are possible?