American Scholastic Mathematics Association Practice Questions

1. Find the units' digit of 17^{17}

the ones digit.

17^1 17^2 17^3 17^4 17^5 17^6

17 289 4913 83521 1419857 24137691

pattern 7 9 3 1 7 9

(7)

2. In base x (x is a positive integer), $35 + 35 + 35 + 35 = 164$. Express x in base 10.

$35_x = 3x + 5$

$164_x = 1x^2 + 6x + 4$

$4(3x+5) = x^2+6x+4$

$12x + 20 = x^2 + 6x + 4$

$0 = x^2 - 6x - 16$

$0 = (x-8)(x+2)$

$x = 8$ $x = -2$

3. If c and d are positive integers, such that $36 + d^2 = c^2$, compute the largest possible value of $c + d$.

$c + d = 36$

$+ \; c - d = 1$

$2c = 37$

$c = 18.5$ ×

$36 = c^2 - d^2$

$36 = (c+d)(c-d)$

6	6
~~36~~	~~1~~
(18)	2
12	3
9	4

$c + d = 18$

$c + d = 18$

$+ \; c - d = 2$

$2c = 20$

$c = 10$ ✓

$c + d = 18$

$10 + d = 18$

$d = 8$ ✓

4. The number 28 (base x) has three times the value of the number 28 (base 10). Find x.

$28_x = 2x + 8$

$28_{10} = 20 + 8$

$28(3) = 2x + 8$

$84 = 2x + 8$

$x = 38$

5. An interior angle of a regular polygon is 165 degrees. Find the number of sides of the polygon.

≅ sides

180 - int. ∠ = ext ∠

180 - 165 = 15

sum of ext ∠ = 360° for all polygons

$\frac{360}{15} = 24$

24 sides

6. If x, $2x + 3$, and $4x$ are the first three terms of an Arithmetic Progression, find the numerical value of the fourth term.

x, $2x+3$, $4x$

+d +d

$(2x+3) - x = 4x - (2x+3)$

$x + 3 = 4x - 2x - 3$

$x + 3 = 2x - 3$

$3 = x - 3$

$x = 6$

6, 15, 24, (33)

+9 +9 +9

7. An integer is represented by a two-digit base ten numeral. If 3 times the sum of its digits is added to the integer, the result is the original integer with its digits reversed. Find all such integers.

x y yx

value of integer: $10x + y$

$3(x+y) + 10x + y = 10y + x$

$3x + 3y + 10x + y = 10y + x$

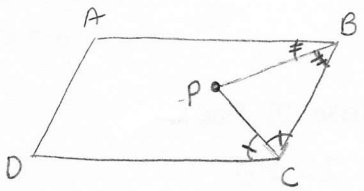
$13x + 4y = 10y + x$

$\frac{12x}{6} = \frac{6y}{6}$

x	y
1	2
2	4
3	6
4	8

or -21, -42, -63, -84

8. In a parallelogram ABCD, the bisectors of interior angles ABC and BCD intersect at P. Find the degree measure of angle BPC.

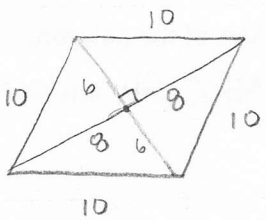


$m\angle ABC + m\angle BCD = 180°$ supplementary

so $m\angle PBC + m\angle PCB = 90°$ complementary

so $m\angle BPC = 90°$

9. The length of a side of a rhombus is 10 and the length of a diagonal is 16. Find the area of the rhombus.



$A = \frac{1}{2} d_1 d_2$ or A = 4 triangles

$A = \frac{1}{2}(16)(12)$ $A = 4\left(\frac{1}{2}(6)(8)\right)$

$A = 96 \text{ units}^2$ $A = 96 \text{ units}^2$