5. Exploring rate of change in other situations | MARS task: Differences

Differences

Lee is working out sequences of numbers.

He starts to construct this table:

Table 1

Position	1st term		2nd term		3rd term		4th term		5th term		6th term
Sequence	1		7		19		37		61		91
1st differences		6		12		18					
2nd differences			6		6						

1. Fill in the gaps in the table.
2. Find the next two terms in the sequence, assuming that the pattern continues.

   7th term _____ 8th term _____

Lee wants to find a formula for the nth term of this sequence.

He knows that it must be of the form $an^2 + bn + c$ where a, b, and c are constants.

So he puts n = 1, 2, 3 and makes Table 2, which he will match with Table 1 to find a, b and c.

Table 2

Position	1st term		2nd term		3rd term
Sequence	$a + b + c$		$4a + 2b + c$		
1st differences		$3a + b$			
2nd differences					

3. Explain why the 3rd term in Table 2 is $9a + 3b + c$.
4. Complete Table 2 to show the first three terms of the sequence and the first and second differences.
5. The formula for the nth term of the sequence in Table 1 is $an^2 + bn + c$.
   By comparing Table 1 and Table 2, Lee can now find a, b and c.
   Write down the values of a, b and c.

   a = _____

b = ______________________________________

c = ______________________________________